

Intertype superconductivity and current-induced self-organization of mixed superconducting states

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Experiment

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Theory

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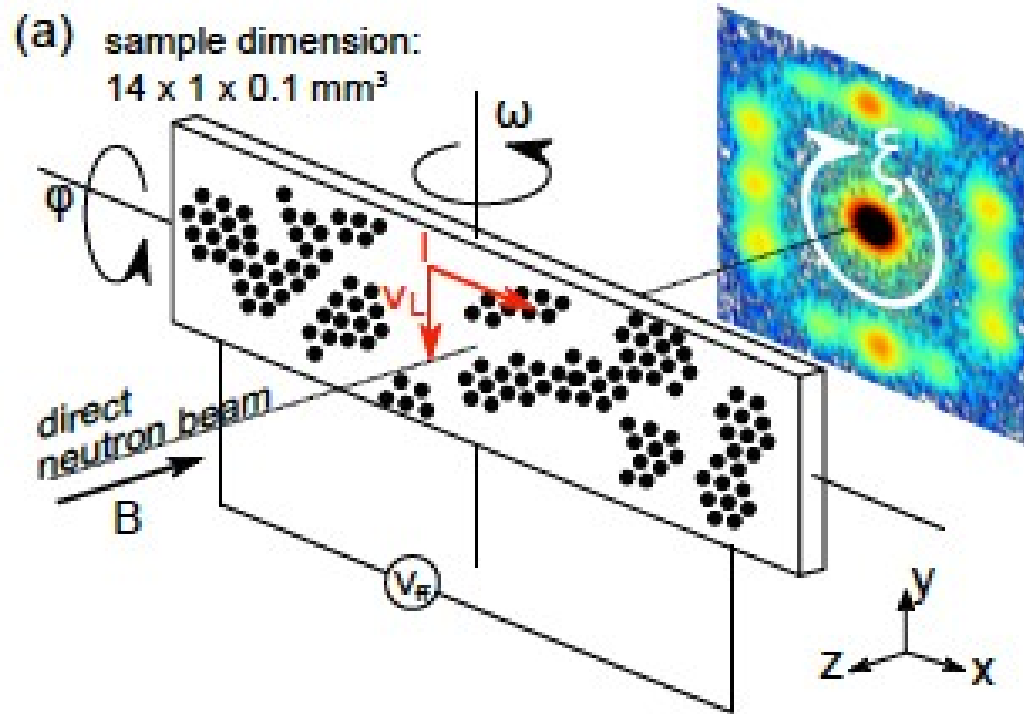
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W. Y. Cordoba-Camacho

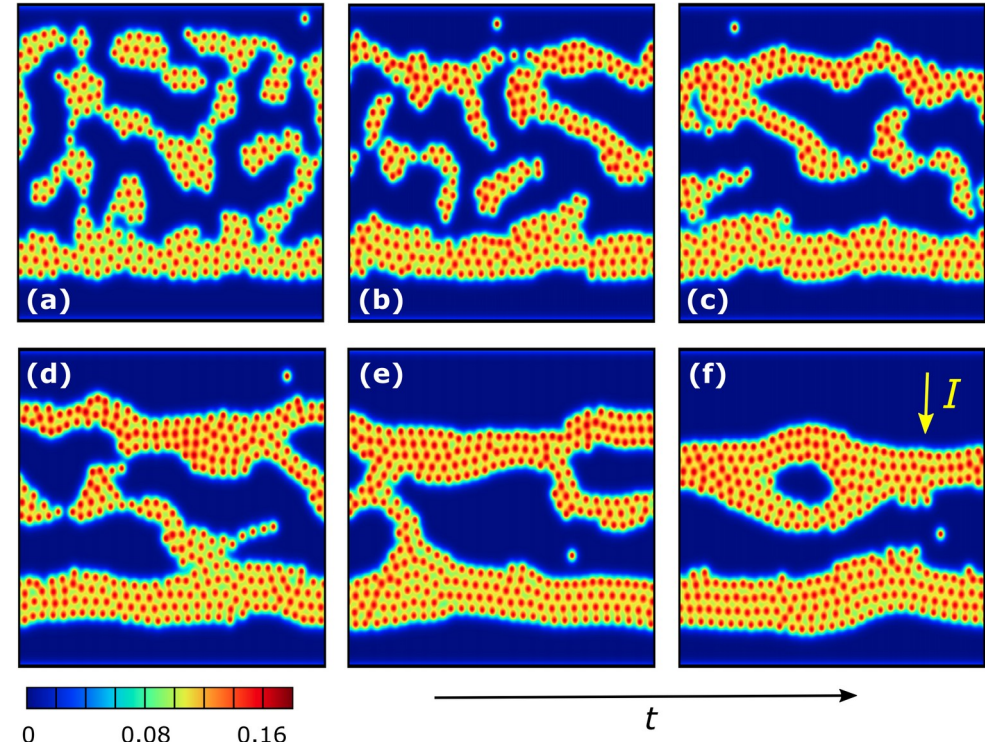
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Evolution of vortex configurations in the intermediate mixed state in Nb

Small angle neutron scattering experiment (SANS) on bulk Nb samples



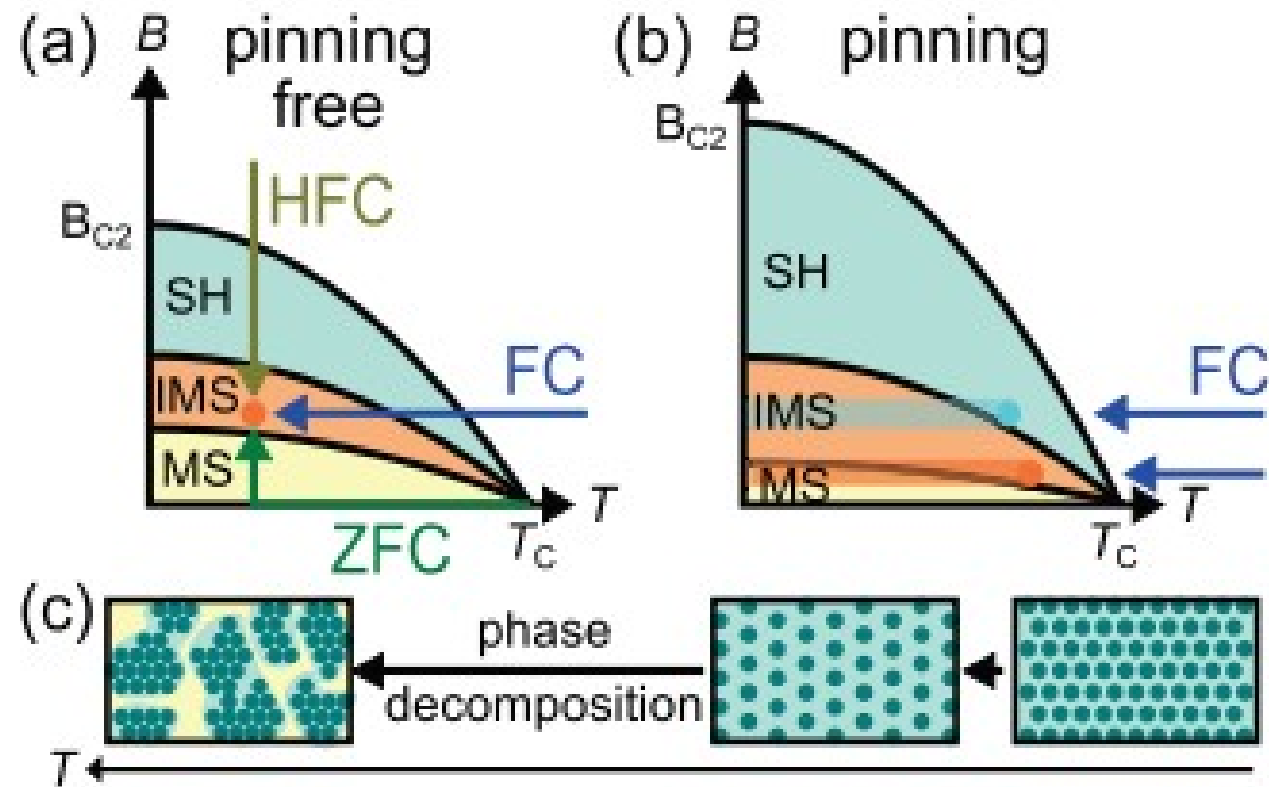
What we believe is going on



1. Evolution starts from vortex clusters (IMS)
2. The IMS structure gradually elongates in the direction perpendicular to the current
3. A superstructure of parallel vortex stripes is formed

Why vortex clusters? Why vortex stipes?

Tentative phase diagram for Nb



Will be discussed

I. Mixed vortex state in low-kappa (type II/1, intertype, IT) superconductors

1. Inter-type superconductivity
2. Vortex structures in the intermediate mix state (IMS)

II. Neutron-scattering experiment

1. Vortex structures and the small-angle neutron scattering (SANS)
2. Manifestation of the IMS state in SANS images
3. IMS distorted by external current

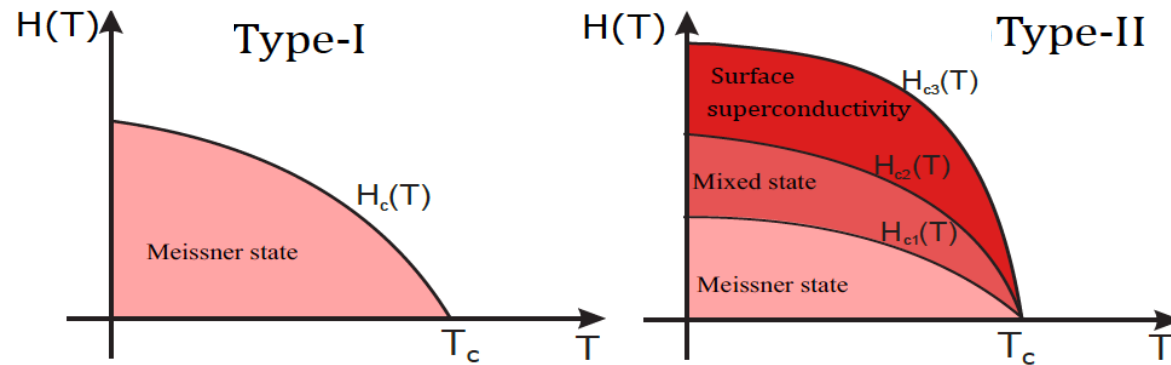
III. Theoretical analysis of the current-driven IMS vortex configurations

1. Choice of the model
2. Results of the simulations

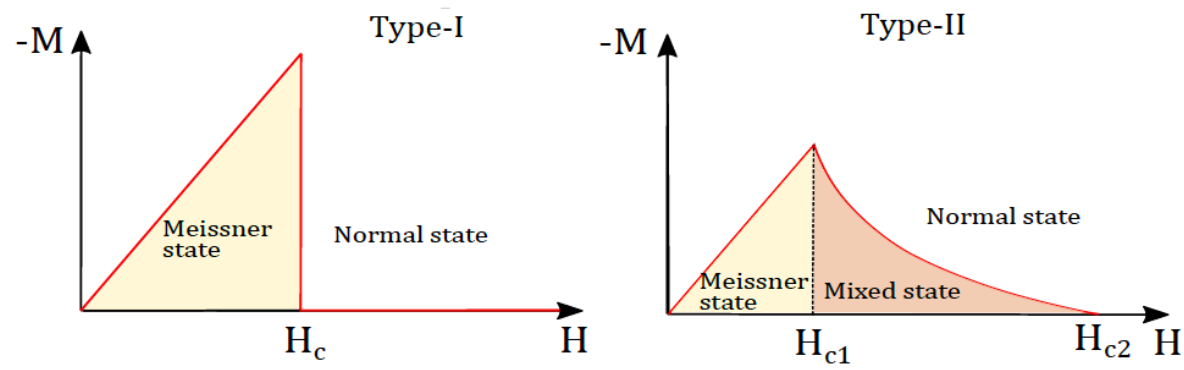
Not exactly in this order – there will be jumps

Two standard superconductivity types (bulk samples)

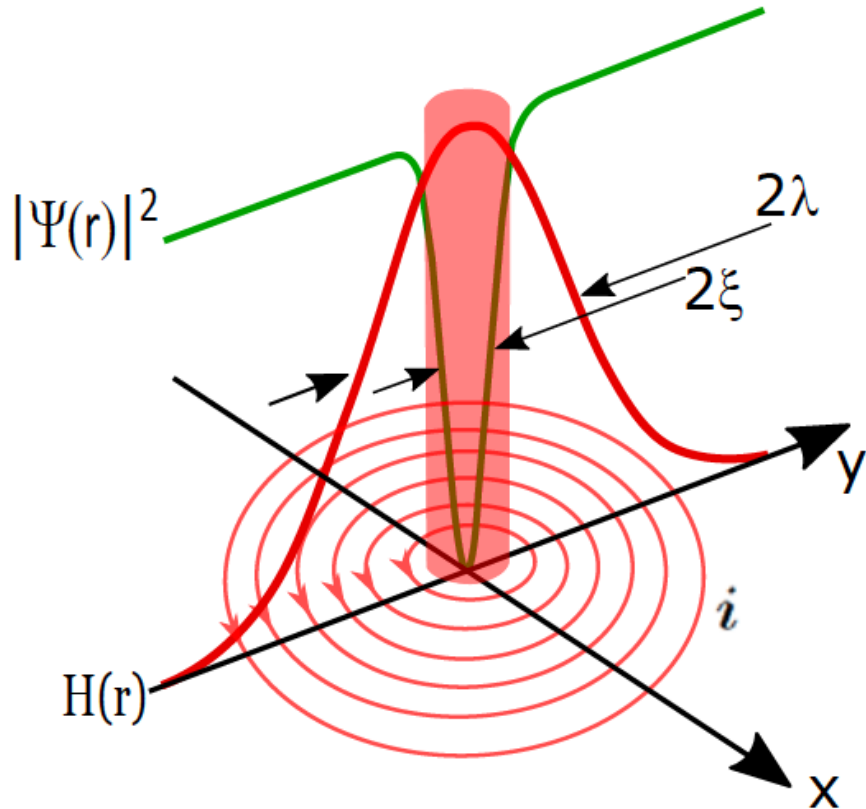
Phase diagram



Magnetization



Magnetic field penetrating a superconductor



2 competing interactions between vortices

Defects in the superconductive condensate – **attractive**

Penetrated magnetic flux - **repulsive**

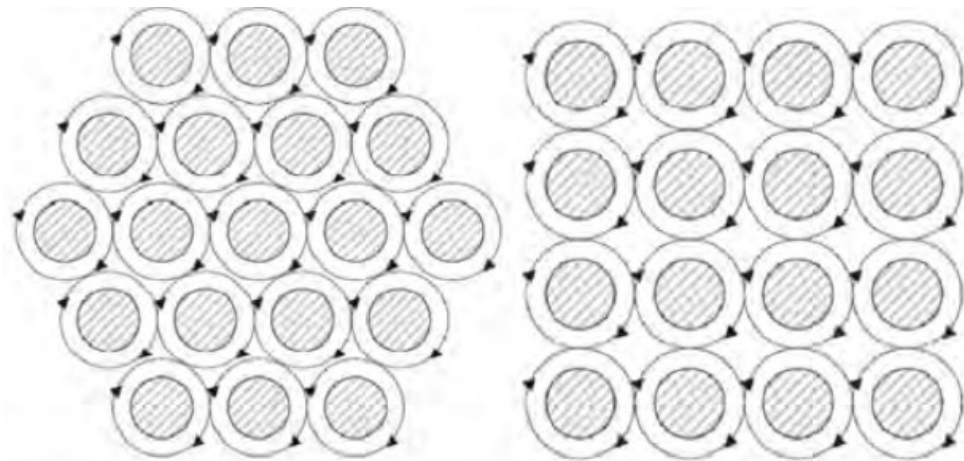
Vortices are repulsive - type II

Vortices are attractive - type I

$$\kappa = \frac{\lambda}{\xi} \gg 1$$

$$\kappa = \frac{\lambda}{\xi} \ll 1$$

Type II superconductors - lattices of repulsive vortices

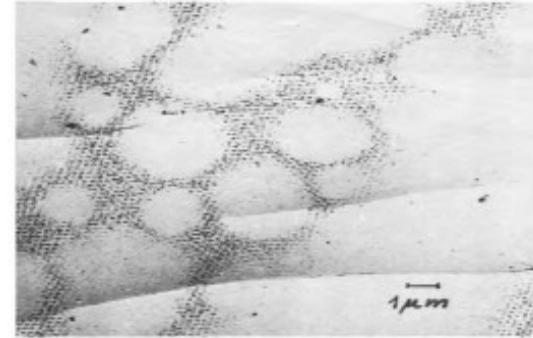
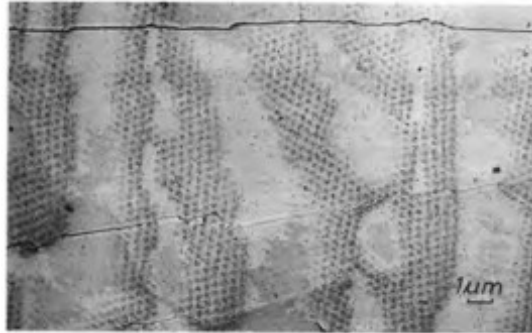


Type I superconductors - attractive vortices collapse

„Low kappa“ superconductors – another type of mixed state

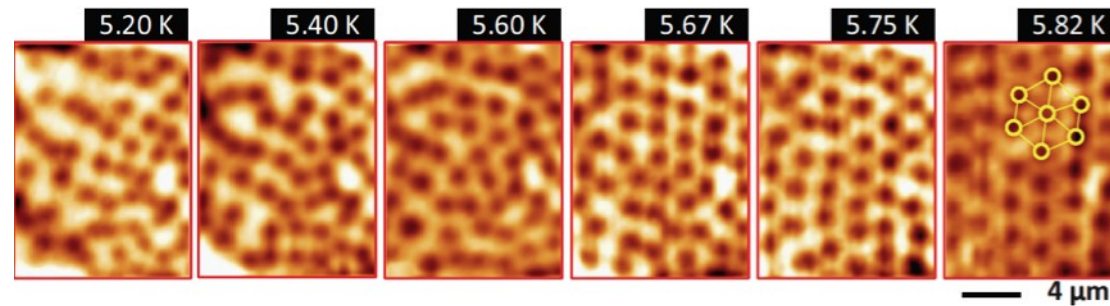
$$\kappa = \frac{\lambda}{\xi} \sim 1$$

Pb-Tl



ZrB12

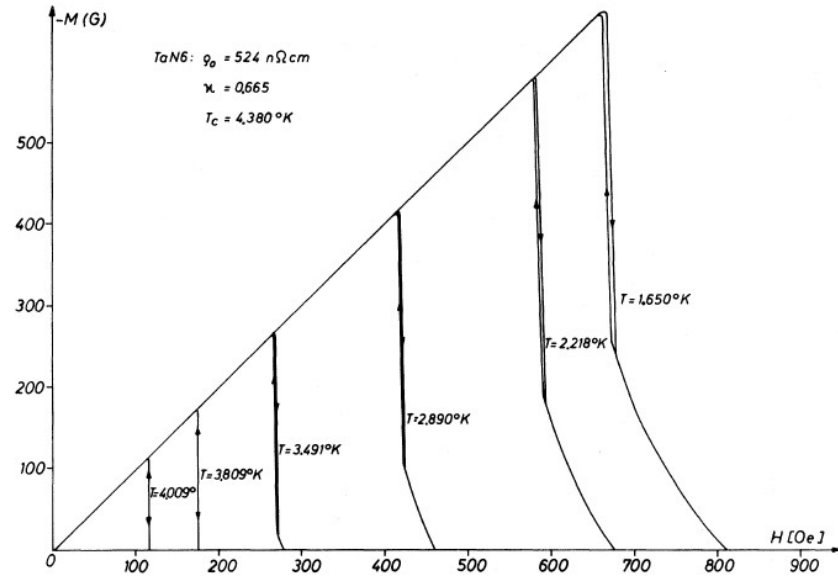
U. Krägeloh, Physics Letters A, 28, 1969.



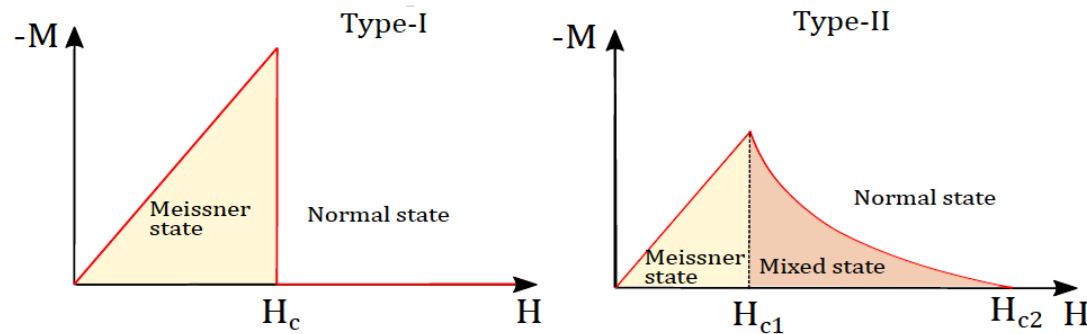
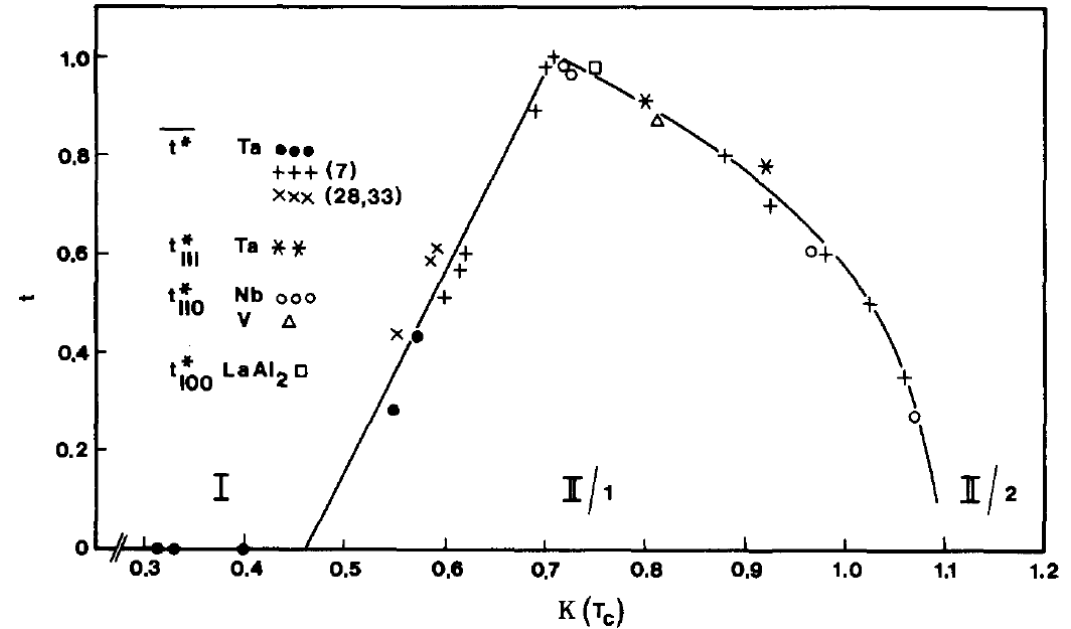
J.-Y. Ge *et al*, Phys. Rev. B **90**, 184511 (2014)

How superconductivity types change

Magnetization



Phase diagram in (T, κ) -plane



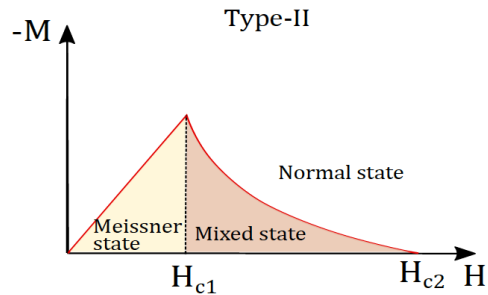
J. Auer and H. Ullmaier, Phys. Rev. B 7, 136 (1973)
H. W. Weber et al., Physica C 161, 272 (1989)

Extended classification of superconductors

Magnetization types

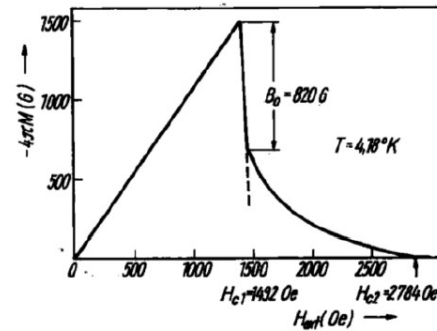
Type II (Type II/2)

$$\kappa \gg \kappa_0$$



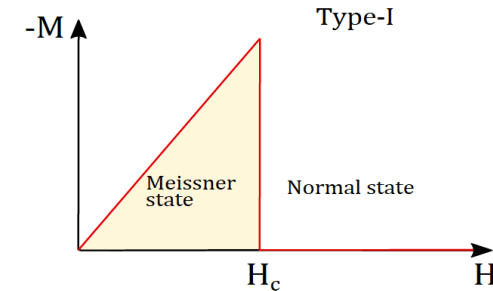
Intertype (Type II/1)

$$\kappa \sim \kappa_0$$



Type I

$$\kappa \ll \kappa_0$$



Vortices

stable
repulsive inter-vortex interaction

stable-unstable,
attractive-repulsive interaction

unstable
attractive inter-vortex interaction

Mixed state configurations

vortex lattice

Vortex clusters, chains, liquid
intermediate mixed state

(Nb,Ta,V...)

no vortices
intermediate state

Questions

1. How describe intermediate mixed state theoretically?

the Ginzburg-Landau theory does not apply, microscopic calculations are too demanding

2. Is the *inter-type* (IT) regime universal?

standard superconductivity types follow from the Ginzburg-Landau theory - Landau theory of second order phase transitions. Is the IT regime depend crucially on details of the microscopic theory or it is based on universal principles and mechanisms?

3. What are essential characteristics of the IT regime?

4. Where can the IT regime be observed?

only in low-kappa materials or elsewhere?

Why GL theory does not work

Ginzburg-Landau equations

$$-\mathcal{K} \nabla^2 \Psi + a \Psi + b |\Psi|^2 \Psi = 0$$

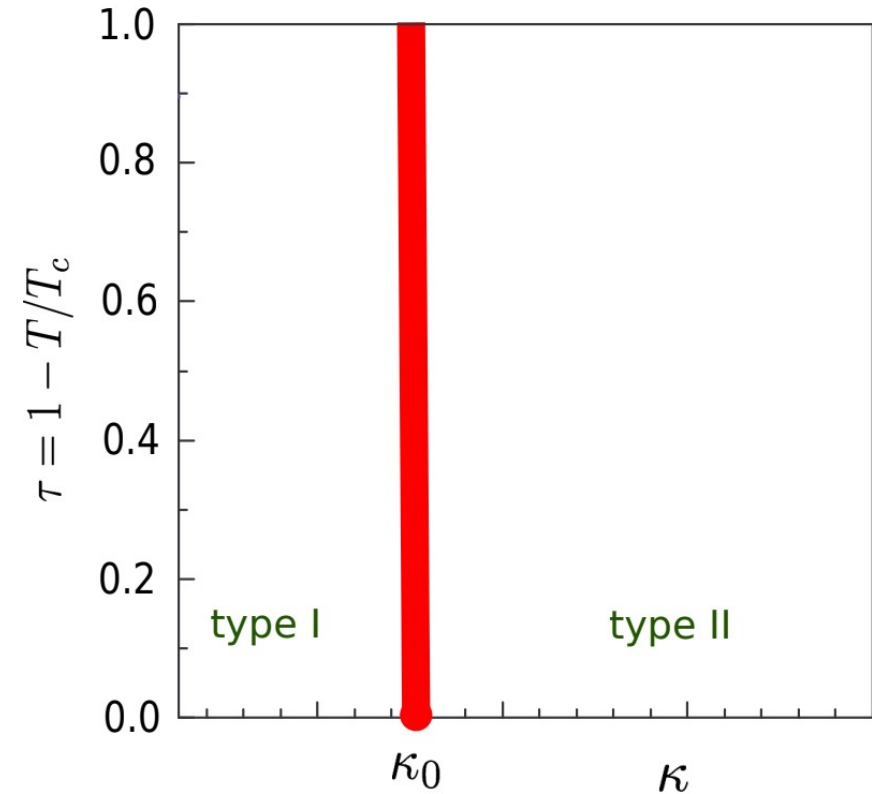
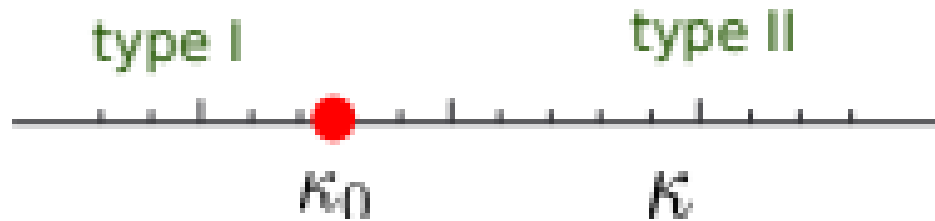
$$\frac{1}{4\pi} \text{rot } \mathbf{B} = \mathcal{K}' \mathbf{j}$$

GL critical point $\kappa_0 = \frac{1}{\sqrt{2}}$

Only one parameter
of the theory

$$\kappa = \frac{\lambda}{\xi}$$

$$\lambda \propto \xi \propto (1 - T/T_c)^{-1/2}$$



GL equations give a single temperature independent critical point, at which the superconductivity type changes abruptly -> GL theory fails to describe the finite IT interval

What happens at the critical point $\kappa_0 = \frac{1}{\sqrt{2}}$

GL equations become *self-dual* Bogomolny (Sarma) equations (in 2D)

$$B = 1 - |\psi|^2 \quad (\partial_y + i\partial_x)\psi = (A_x - iA_y)\psi$$

Solution structure is similar to that of the Landau ground state (charged particle in magnetic field)

$$\psi = e^{-\theta} \phi$$

$$(\partial_x^2 + \partial_y^2)\theta = B \quad (\partial_x + i\partial_y)\phi = 0$$

$$B = 1 - e^{-2\theta}|\phi|^2$$

Solution is defined via an arbitrary analytic function of the complex variable

N-vortex solution

$$\phi_N = \prod_{i=1}^N (z - a_i) \quad a_i \text{ - vortex positions}$$

Degeneracy of the GL theory

Gibbs energy

$$G(\text{mixed state}, H = H_c) = G(\text{Meissner state}, H = H_c)$$

Infinite degeneracy at the critical (Bogomolny) point

All vortex configurations have the same energy

Energy does not depend on vortex positions – vortices do not interact

Suggestion: lifting the degeneracy creates a finite crossover (IT) interval between superconductivity types

Perturbation expansion for the BCS theory

Perturbation expansion of the microscopic equations (Bogoljubov - de Gennes, Gorkov, Eilenberger...)

$$\Delta = \tau^{1/2}(\Psi + \tau\psi + \dots), \quad \mathbf{B} = \tau(\mathfrak{B} + \tau\mathbf{b} + \dots), \quad \mathbf{r} \rightarrow \tau^{-1/2}\mathbf{r}, \quad \nabla \rightarrow \tau^{1/2}\nabla$$

gap function magnetic field coordinate scaling

$$\tau = 1 - T/T_c$$

Perturbation expansion for the free energy $\mathfrak{f} = \tau^2(\mathfrak{f}^{(0)} + \tau\mathfrak{f}^{(1)} + \dots)$,

GL contribution $\mathfrak{f}^{(0)} = \frac{\mathfrak{B}^2}{8\pi} + a|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \mathcal{K}|\mathfrak{D}\Psi|^2$

Leading correction

$$\begin{aligned} \mathfrak{f}^{(1)} = & \frac{a}{2}|\Psi|^2 + 2\mathcal{K}|\mathfrak{D}\Psi|^2 + b|\Psi|^4 + \frac{b}{36}\frac{e^2\hbar^2}{m^2c^2}\mathfrak{B}^2|\Psi|^2 - \mathcal{Q}\left\{|\mathfrak{D}^2\Psi|^2 + \frac{1}{3}(\text{rot } \mathfrak{B} \cdot \mathbf{i}) + \frac{4e^2}{\hbar^2c^2}\mathfrak{B}^2|\Psi|^2\right\} \\ & - \frac{\mathcal{L}}{2}\left\{8|\Psi|^2|\mathfrak{D}\Psi|^2 + [\Psi^2(\mathfrak{D}^*\Psi^*)^2 + \text{c.c.}]\right\} - \frac{c}{3}|\Psi|^6 \end{aligned}$$

where the order parameter and the field satisfy the GL equations

$$-\mathcal{K}\mathfrak{D}^2\Psi - a\Psi + b|\Psi|^2\Psi = 0, \quad \frac{1}{4\pi}\text{rot}\mathfrak{B} = \mathcal{K}\mathbf{i}, \quad \mathfrak{D} = \nabla - i2e/\hbar c\mathbf{A}, \quad \mathbf{i} = i\frac{2e}{\hbar c}(\Psi\mathfrak{D}^*\Psi^* - \Psi^*\mathfrak{D}\Psi),$$

Final expressions for the energy

Gibbs free energy of an (arbitrary) multi-vortex configuration

$$\frac{\mathcal{G}}{\tau^2} = -\mathcal{I} \frac{\delta\kappa}{\kappa_0} + (C_{\mathcal{I}}\mathcal{I} + C_{\mathcal{J}}\mathcal{J})\tau \quad \mathcal{I} = \int |\Psi|^2 (1 - |\Psi|^2) d\mathbf{r} \quad \mathcal{J} = \int |\Psi|^4 (1 - |\Psi|^2) d\mathbf{r}$$

Ψ - multi-vortex (dimensionless) solution of the GL equations, obtained for arbitrary vortex positions

$$B = 1 - |\Psi|^2 \quad (\partial_y + i\partial_x)\Psi = (A_x - iA_y)\Psi$$

$C_{\mathcal{I}}, \quad C_{\mathcal{J}}$ are constants, obtained from the microscopic model for the carrier states

In the vicinity of the Bogomolny point the leading order corrections to the GL theory depends only on the solutions to the GL theory.

This dependence is universal! The material parameters enter only via the coefficients $C_{\mathcal{I}}, \quad C_{\mathcal{J}}$

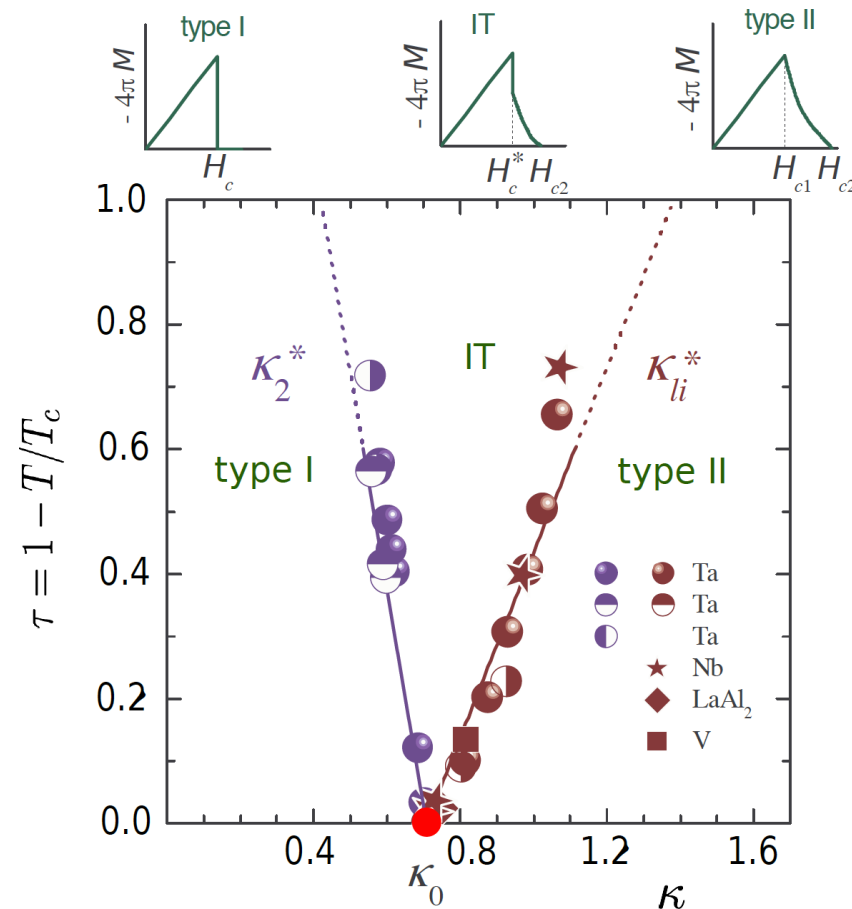
Boundaries of the IT domain

Theory result - BCS superconductor, s-wave pairing, spherical Fermi surface the result is independent of the microscopic parameters

the line at which the vortices become attractive $\kappa_{li}^* = \kappa_0(1 + 0.95\tau)$

the line at which the mixed state disappears $\kappa_2^* = \kappa_0(1 - 0.407\tau)$

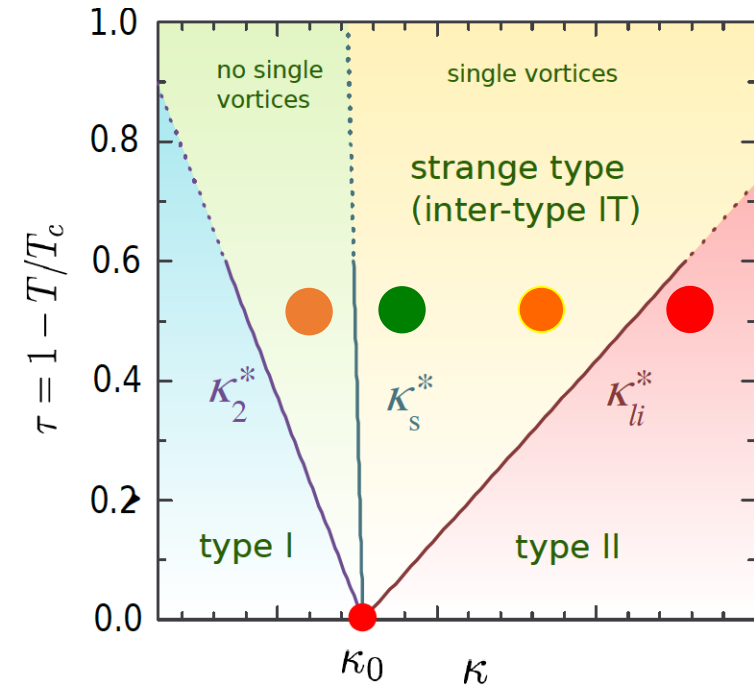
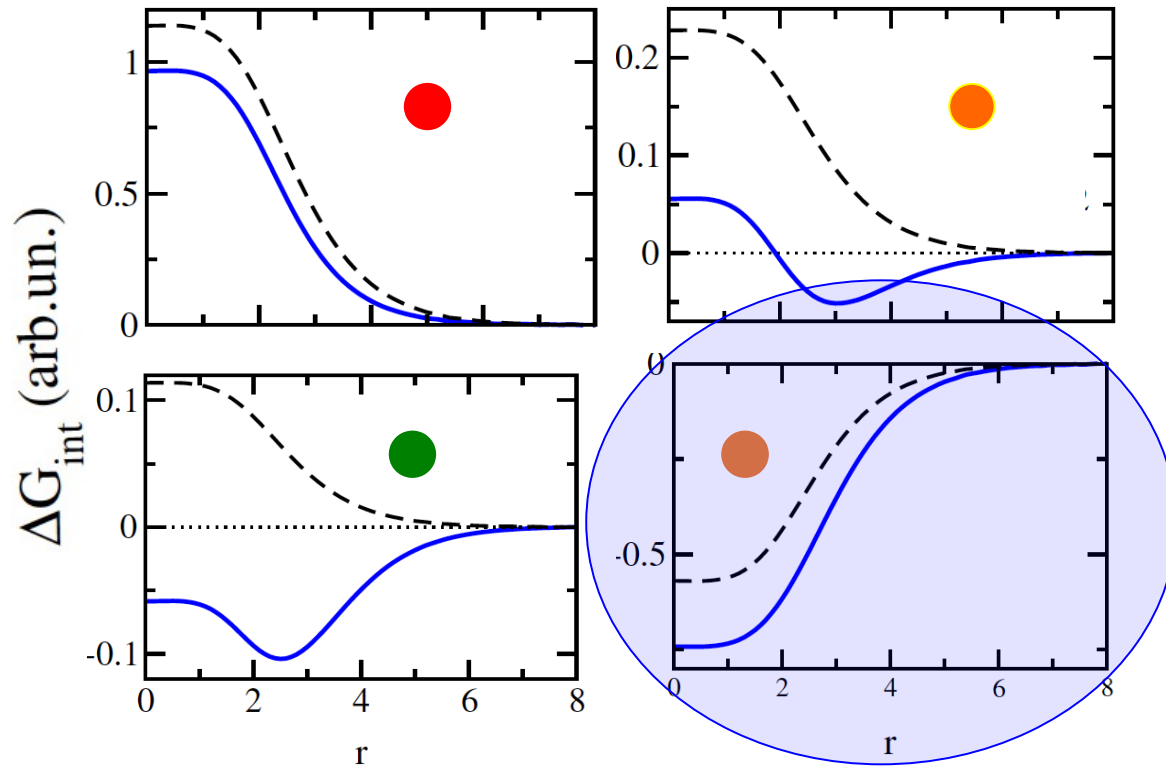
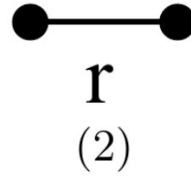
Boundaries for the IT domain from the magnetization curves



- U. Krageloh, Phys. Lett. A 28, 657 (1969)
- U. Essmann, Physica 55, 83 (1971)
- D. R. Aston et al, Phys. Rev. B 3, 2231 (1971)
- U. Kumpf, Phys. Status Solidi (b) 44, 557 (1971)
- J. Auer and H. Ullmaier, Phys. Rev. B 7, 136 (1973)
- H. W. Weber et al., Physica C 161, 272 (1989)

Vortex-vortex interaction

Interaction potential vs distance



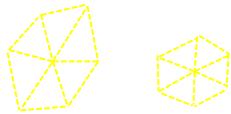
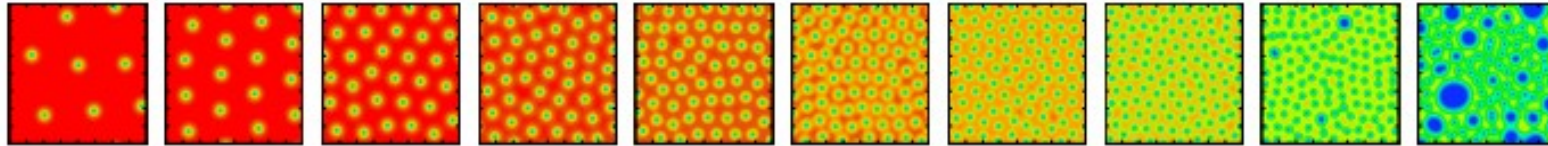
Vortices are unstable but the mixed state exists!

Vortex configurations

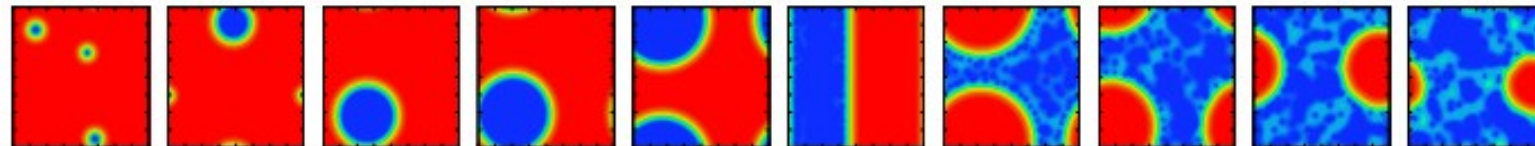
H (field)



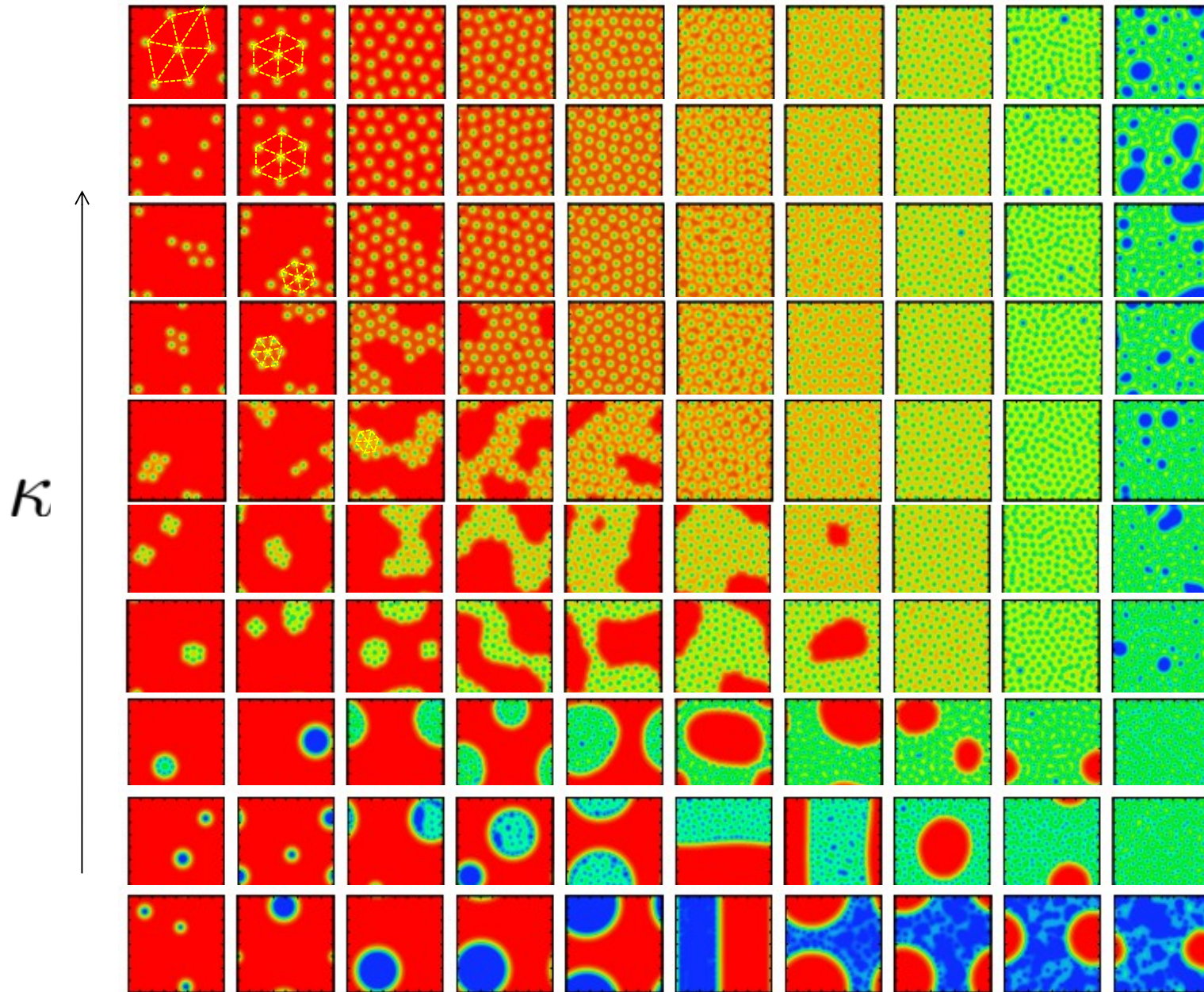
Type II (vortex lattice)



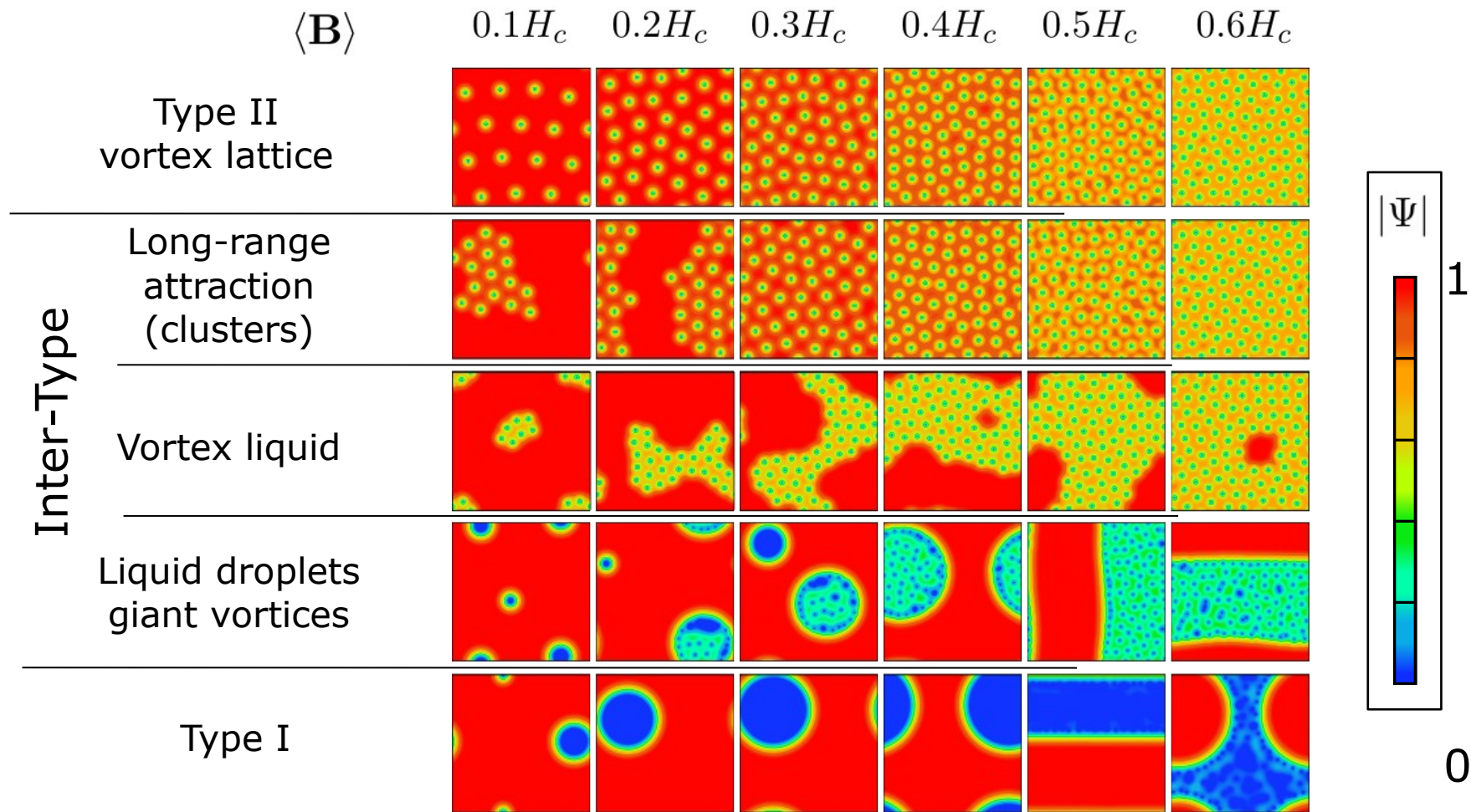
Type I (lamellas)



Vortex configurations in the Type-I/Type-II crossover

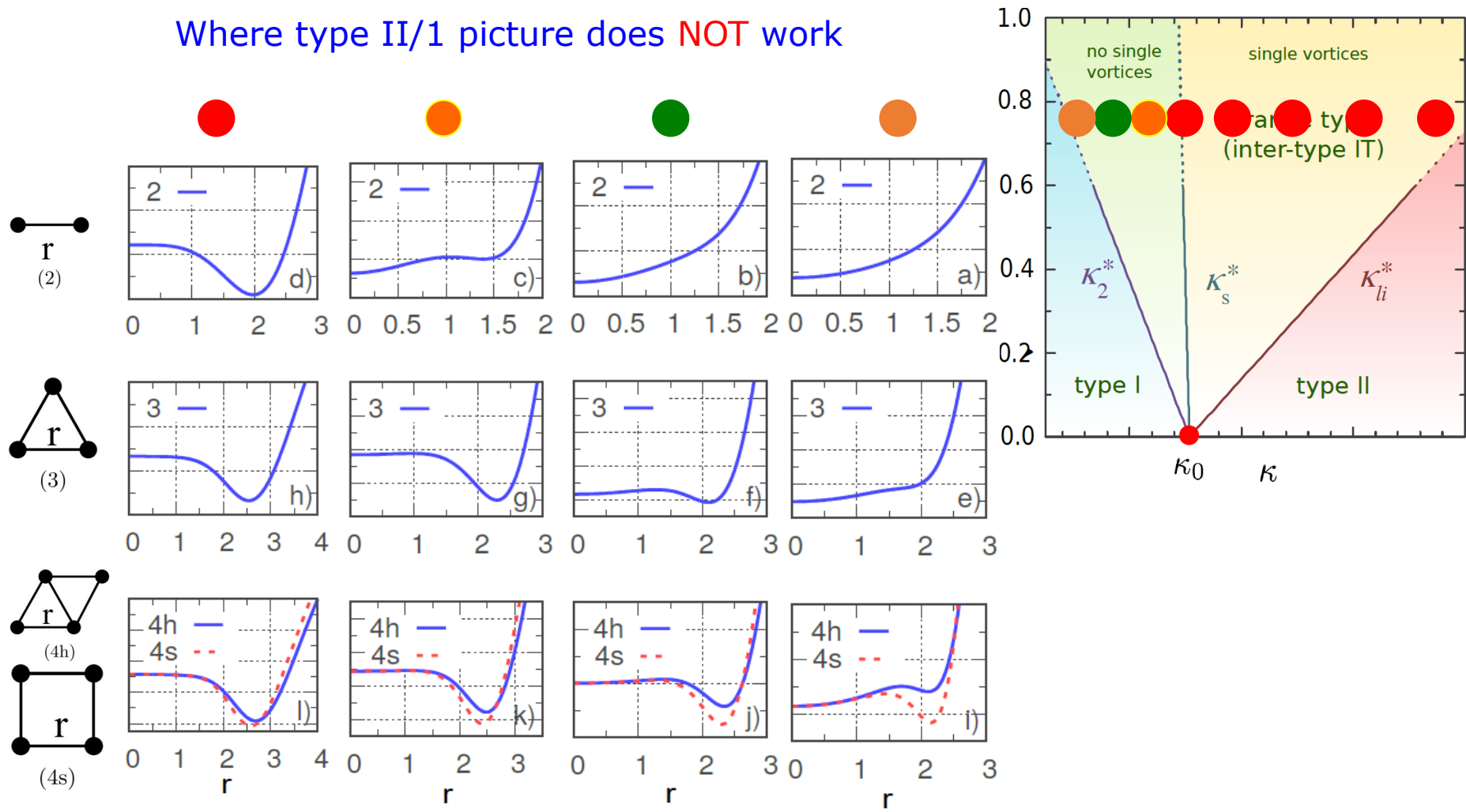


Vortex configurations

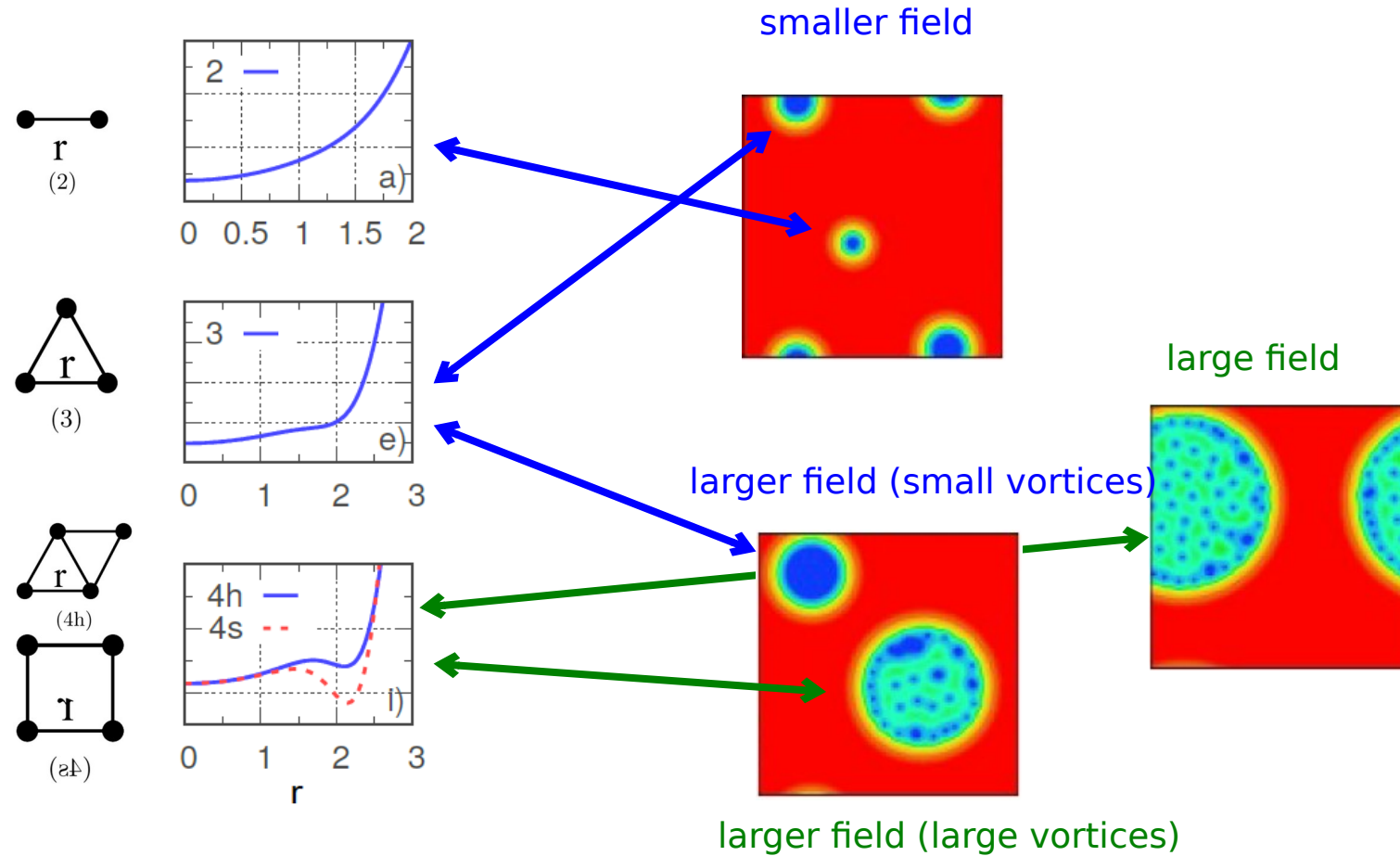


Multi-vortex interactions

Where type II/1 picture does **NOT** work



Vortex interactions & vortex configurations

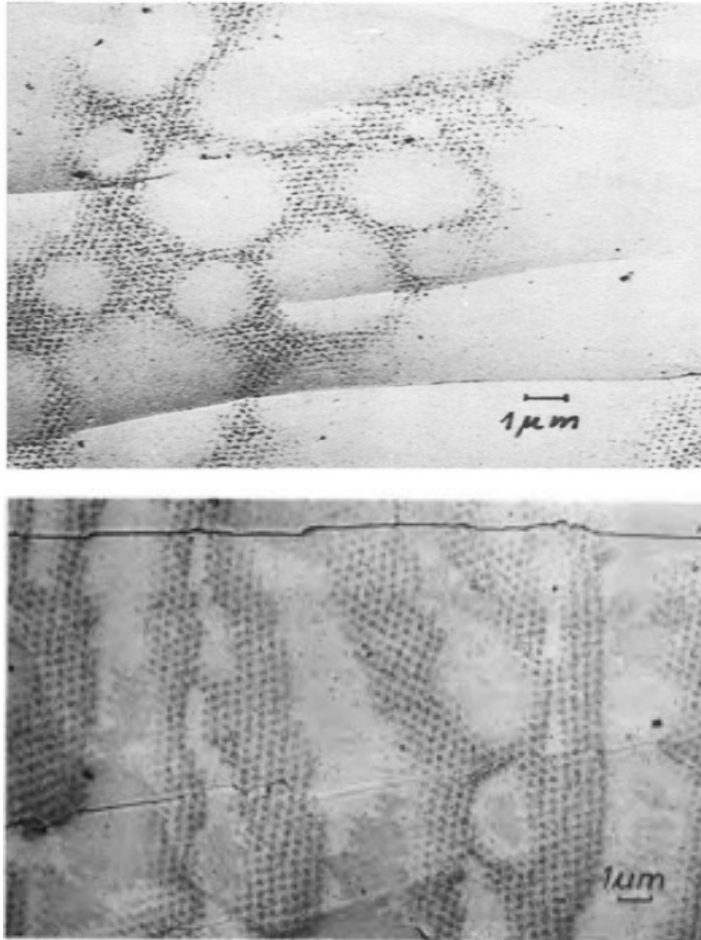


S. Wolf et al, Phys Rev B 96, 144515 (2017)

A. Vagov et al, 2018

Typical vortex configurations in the intermediate mixed state

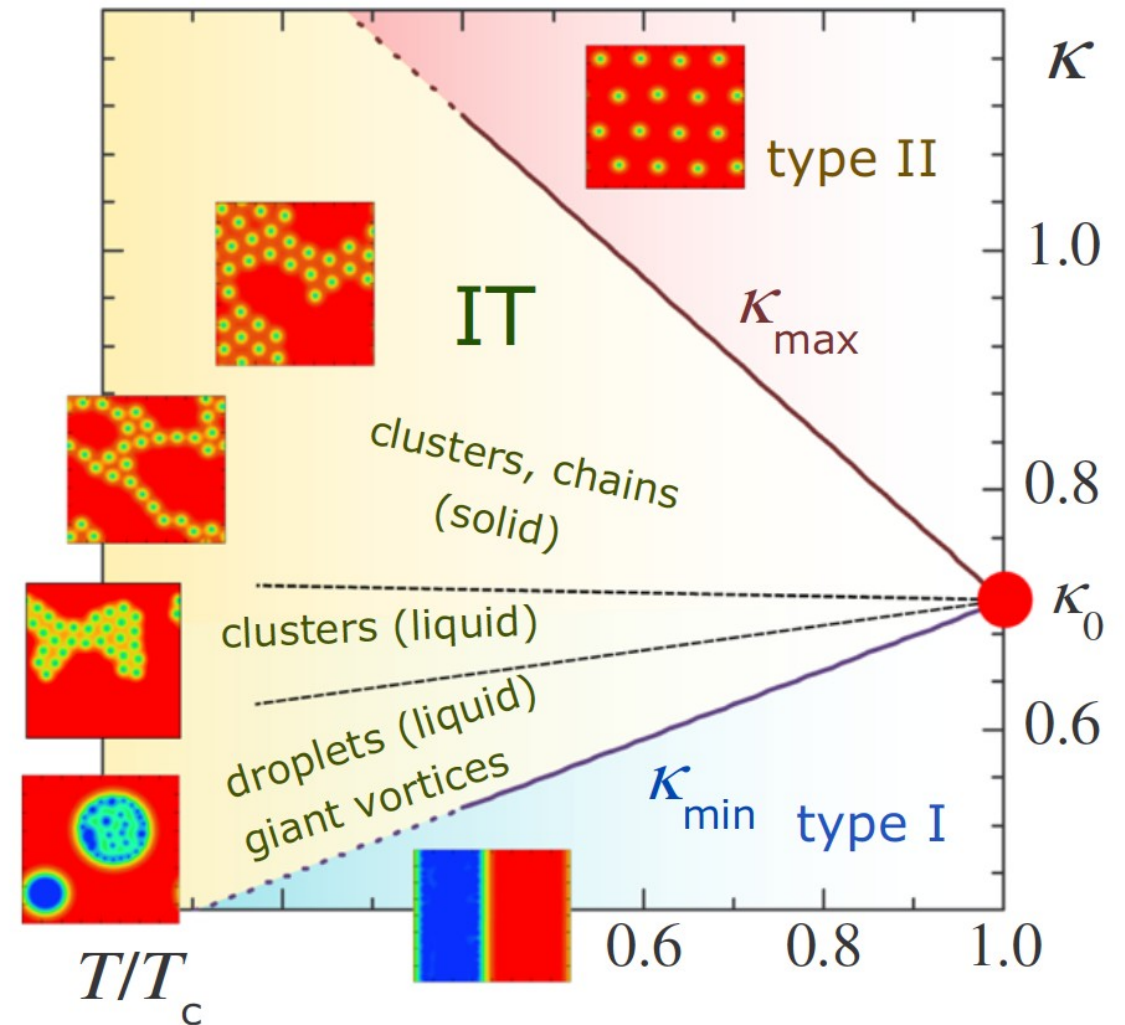
IMS configurations in low- κ materials



U. Krägeloh, Phys. Lett. A 28, 1969

E. H. Brandt and M. P. Das, J. Supercond. Nov. Magn. 24, 57 (2011)

IMS configurations in theoretical simulations



A. Vagov et al., Commun. Phys. 3, 58 (2020)

Type-I/Type-II crossover and IMS in thin films and wires

The IT crossover regime takes place due to a mechanism of removing the degeneracy at the B-point.

This is a very general mechanism that can be expected in many superconductive materials & devices.

Thin films and wires offer an example of this type. When a film made of a type I material, vortices inside are fully attractive and unstable. Stray magnetic field outside a sample makes vortex interaction repulsive. The repulsion dominates for an ultra-thin film making it a type II superconductor.

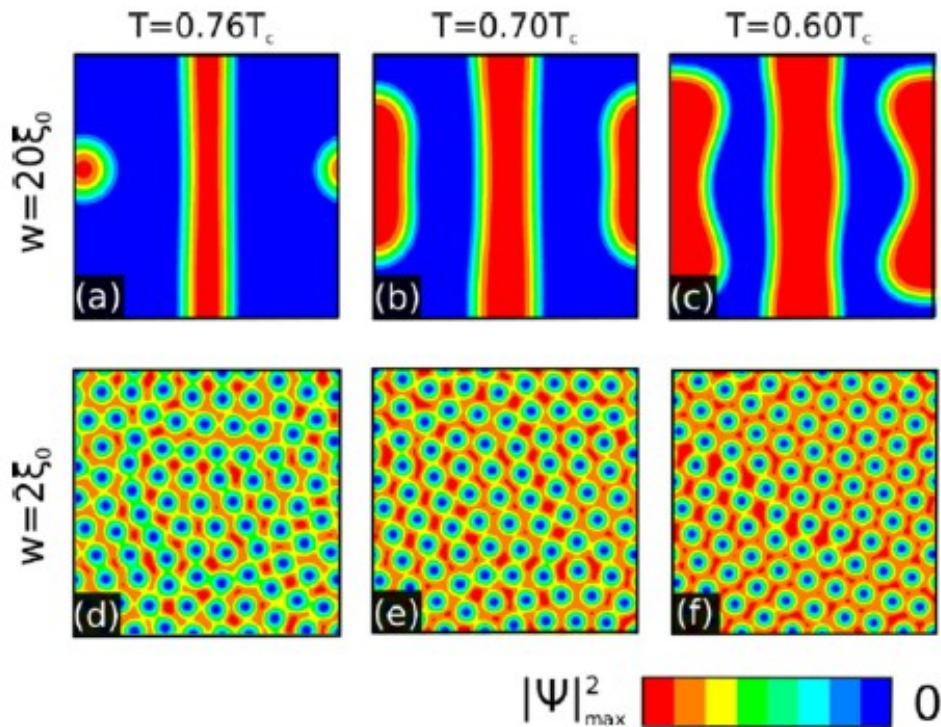
When crossing between the type I and II the film undergoes the IT regime, where the vortex interaction is neither fully repulsive nor attractive.

Type I/II crossover in thin films and wires

Exotic vortex configurations in the IT regime are related to the degeneracy of the B-point. They can also be seen as a result of a competition between different length scales in the vortex interaction – attraction and repulsion.

Thin films and wires offer an example where the competition of different length scales in the interaction takes place.

In a type I material vortices are attractive. The stray magnetic field outside a film makes vortices repulsive. The repulsion dominates for ultra-thin films making it a type II superconductor.



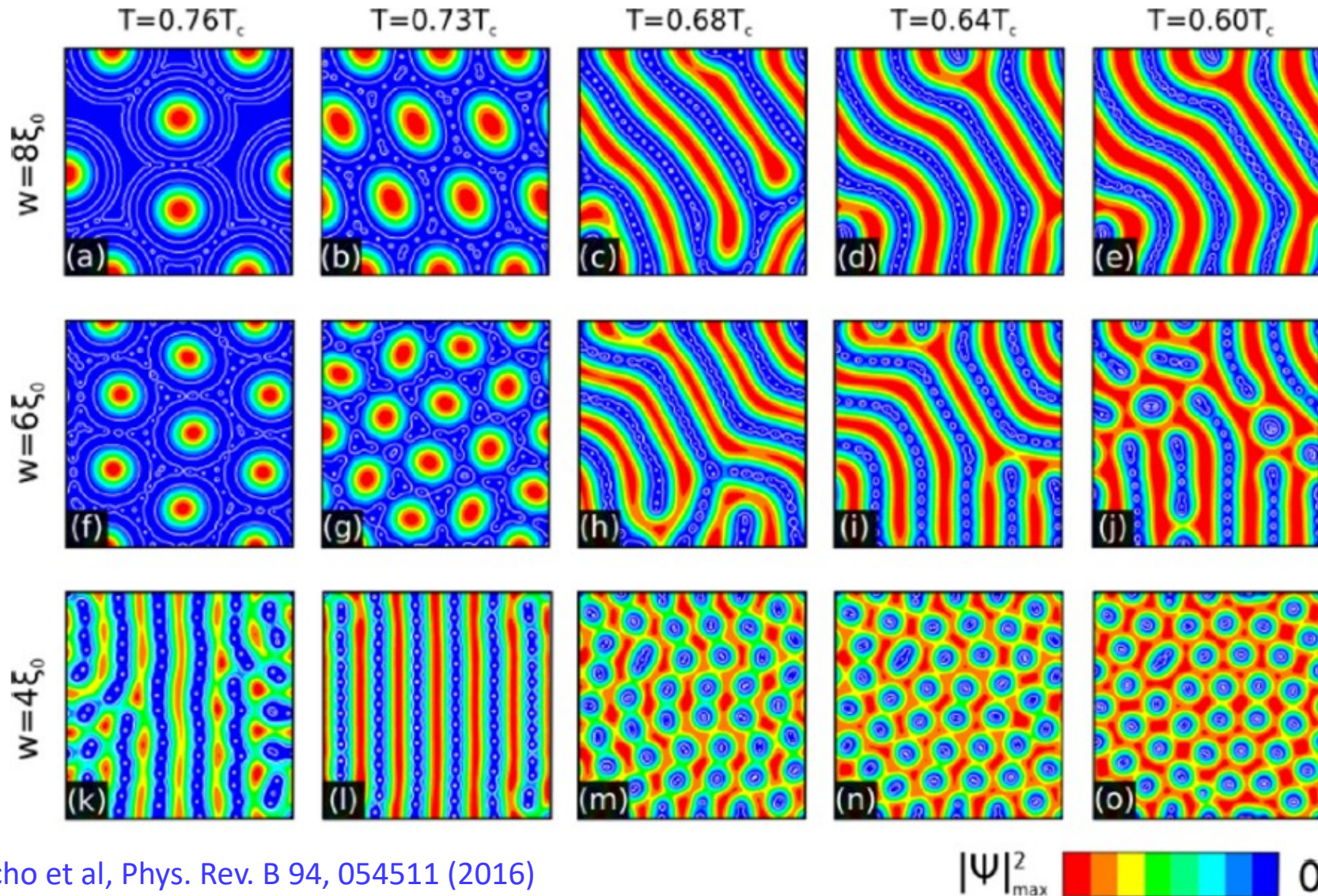
While crossing between the type I and II the film undergoes the IT regime, where the vortex interaction is neither fully repulsive nor attractive.

Superconductive film, GL theory + Maxwell equations

W. Y. Cordoba-Camacho et al, Phys. Rev. B 94, 054511 (2016)

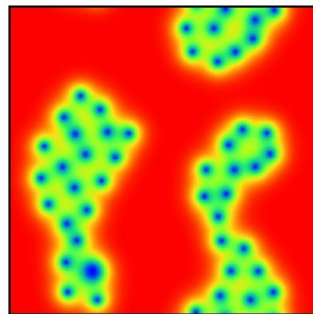
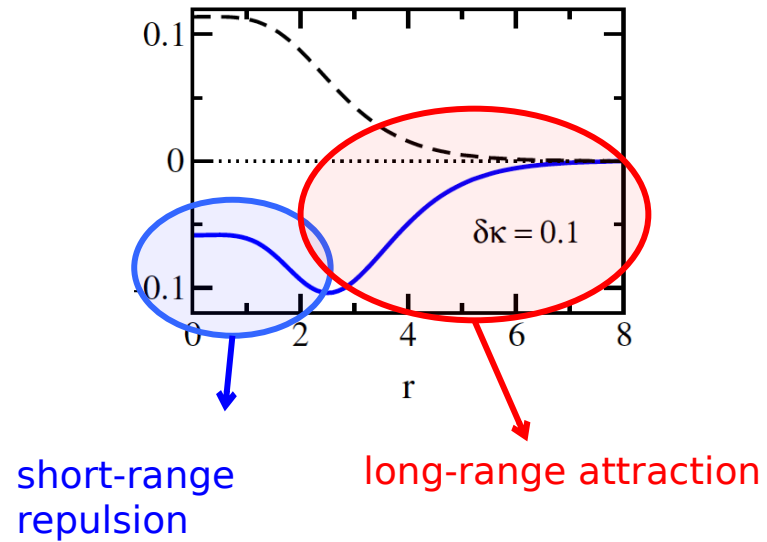
Vortex configurations in thin superconductive films

The width of the sample is an additional parameter that controls the type crossover

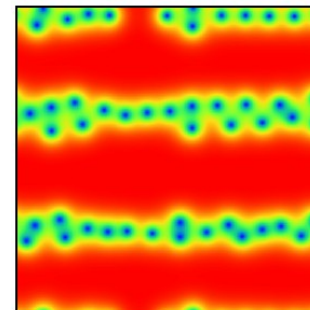
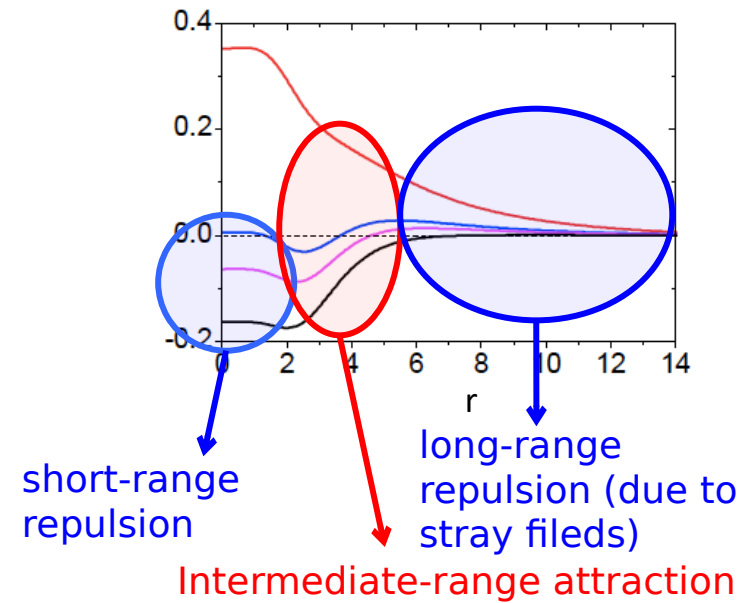


The role of the stray fields in thin superconductive films

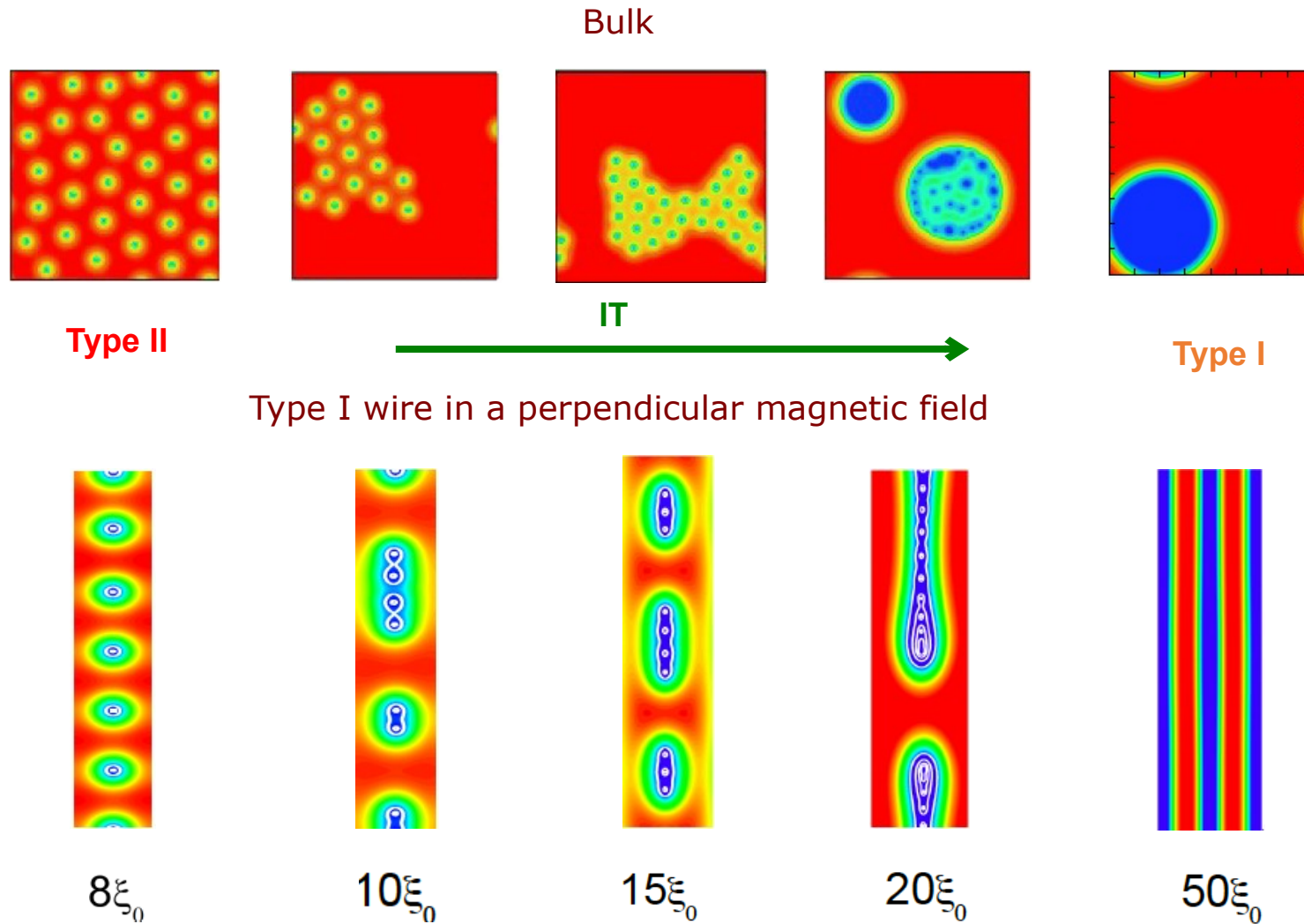
IT regime in bulk samples



IT regime in films (different widths)



Vortex configurations in bulk and in thin wires



Multi-band (muti-gap) superconductors

A classical example is MgB2 with two gaps.

[V. V. Moshchalkov et al, Phys.Rev.Lett.102, 117001 \(2009\).](#)

A spatial profile of the superconducting state in those materials can be described theoretically using a GL-like model with two gap functions and linear Josephson coupling:

$$f = \sum_{\nu=1,2} \left(\frac{1}{2m_{\nu}} |\mathbf{D}\Psi_{\nu}|^2 + \alpha_{\nu} |\Psi_{\nu}|^2 + \frac{\beta_{\nu}}{2} |\Psi_{\nu}|^4 \right) - \Gamma \{ \Psi_1^* \Psi_2 + \Psi_1 \Psi_2^* \} + \frac{\mathbf{B}^2}{8\pi}$$

It is sometimes claimed that existence of 2 bands leads to a novel superconductivity type (“1.5 type”)

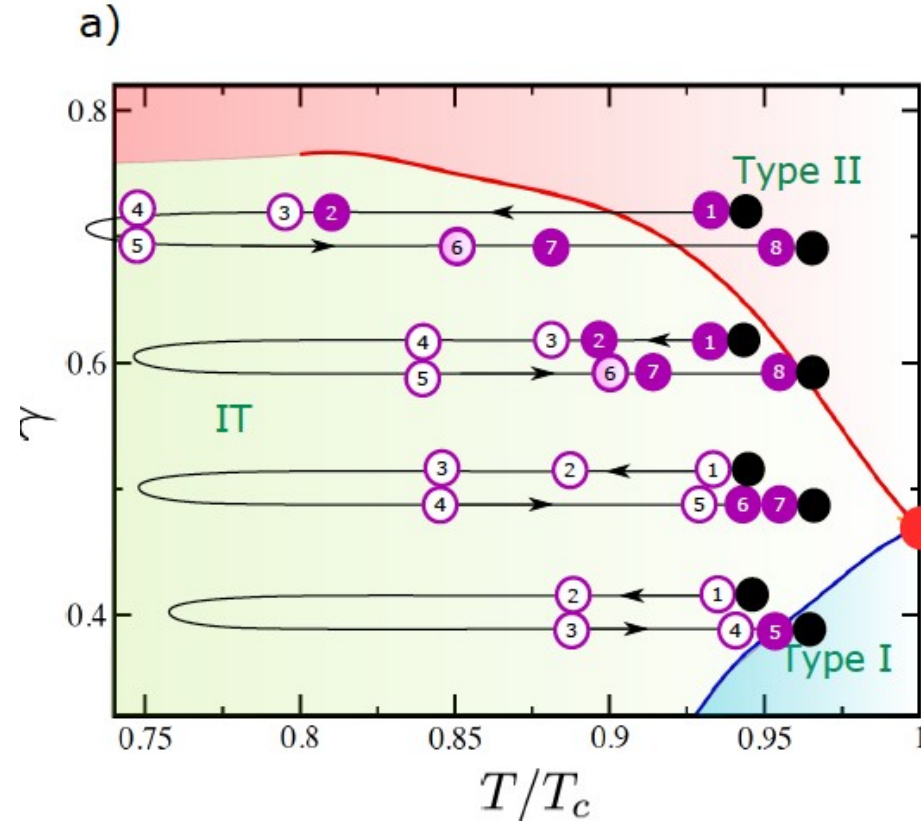
[E. Babaev et al., Phys. Rev. Lett. 105, 067003 \(2010\)](#)

Others had an opinion that the existence of 2 bands makes not much difference with single-band superconductors

[V. G. Kogan and J. Schmalian, Phys. Rev. B 83, 054515 \(2011\), A.A. Shanenko et al, Phys. Rev. Lett. 106, 047005 \(2011\)](#)

2-band superconductor – multi-component GL model

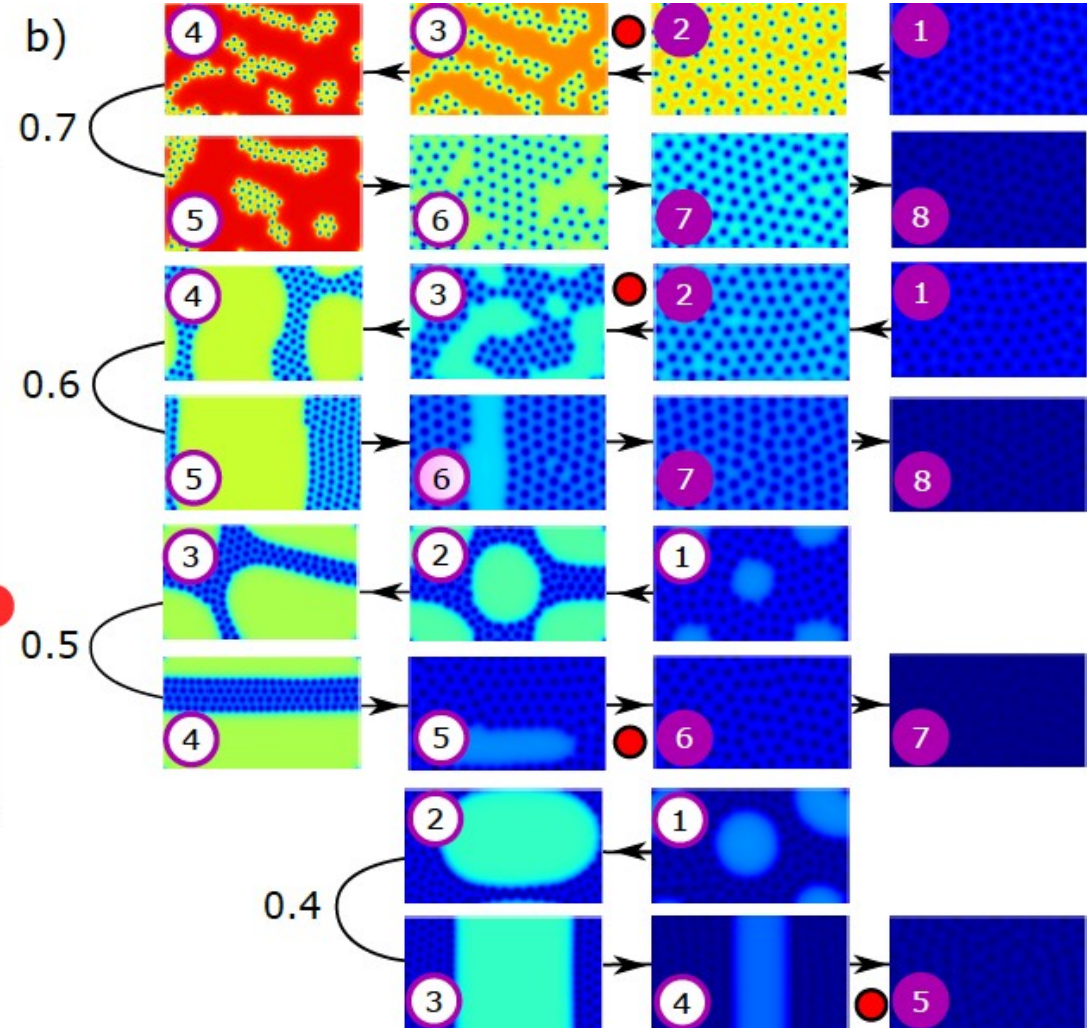
Phase diagram: red line – vortices become attractive at long range, blue line – mixed state (vortices) disappears.



Vortex interaction in the intermediate area is non-monotonic, with a large multi-vortex component.

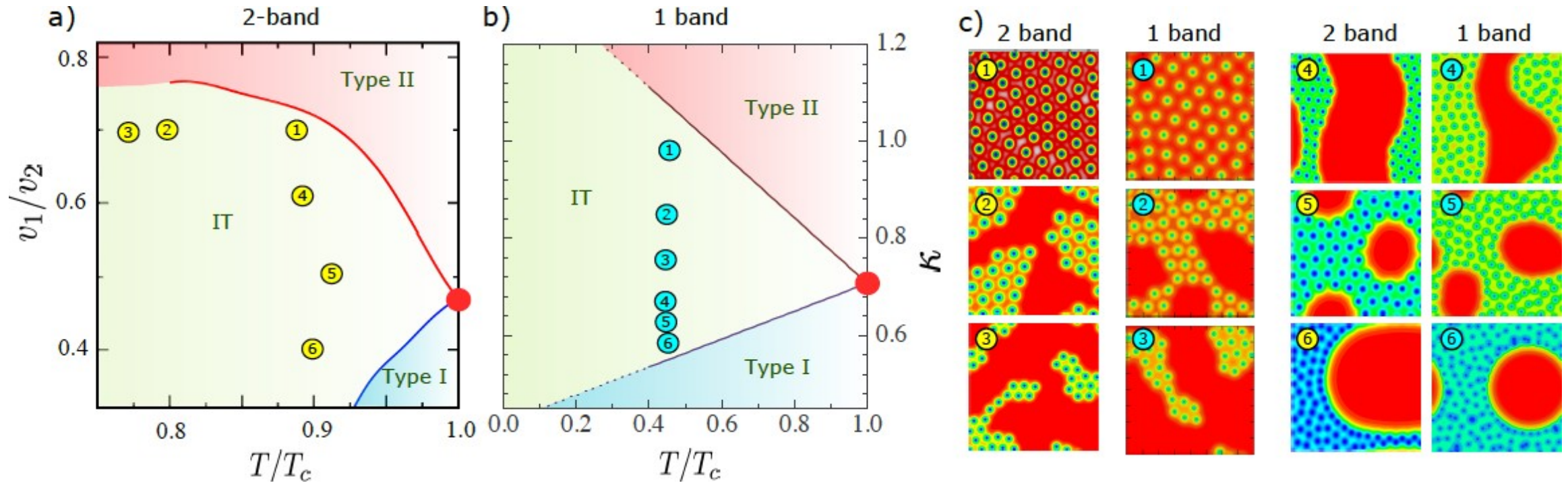
R. M. da Silva et al, Scientific Reports 5, 12695 (2015)

Vortex configurations



E. V. d. S. Junior et al. unpublished (2022)

2-band vs 1-band superconductor



In the range of applicability of the GL theory, the results for the 2-band model are qualitatively similar to those for 1-band superconductors.

Answers to the questions

- How can one describe the *inter-type* (IT) regime theoretically?

Perturbation expansion in the vicinity of the B-point yields a very satisfactory description

- Is the *inter-type* (IT) regime a universal phenomenon, governed by a fairly general mechanism?

Yes. In bulk samples it is related to the infinite degeneracy of the superconductive state at the self-dual Bogomolny point. The degeneracy is lifted in the vicinity of this point creating a finite domain of Intertype superconductivity with the intermediate mixed state of exotic vortex configurations.

Possible vortex structures can be affected by the degeneracy lifting mechanism, however, they are very similar in different structures.

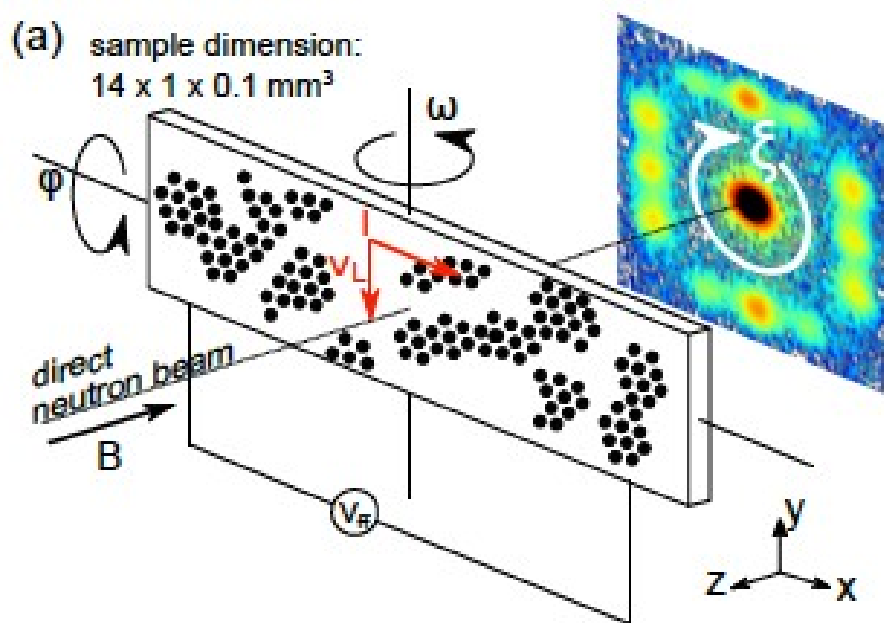
The IT regime is also characterized by a competition of several length scales in the vortex interaction.

- Where can the crossover IT regime be observed?

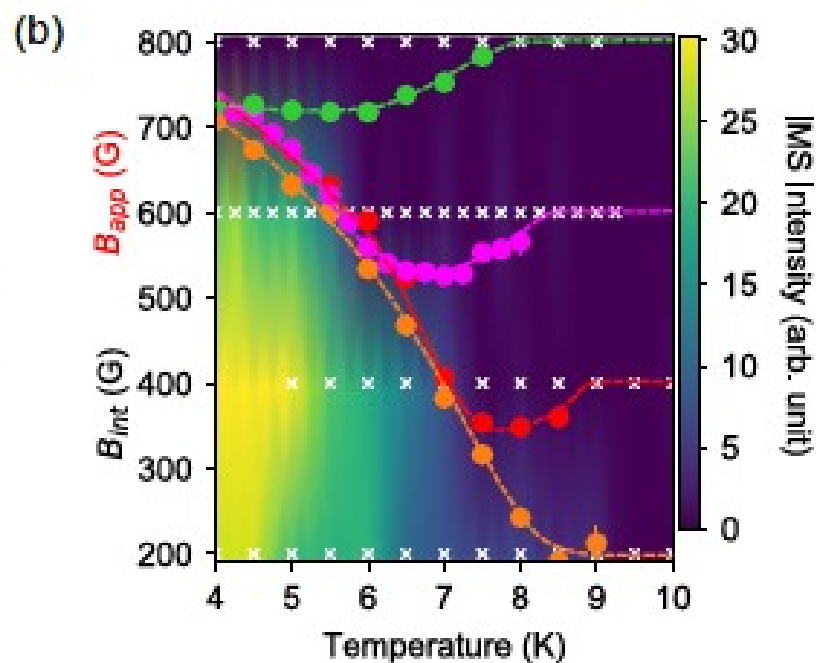
1. Standard low-kappa BCS superconductors
2. Many-band superconductors
3. Thin films
4. Magnetic superconductors
5.

Small angle neutron scattering experiment - SANS

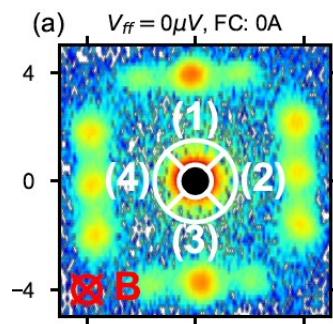
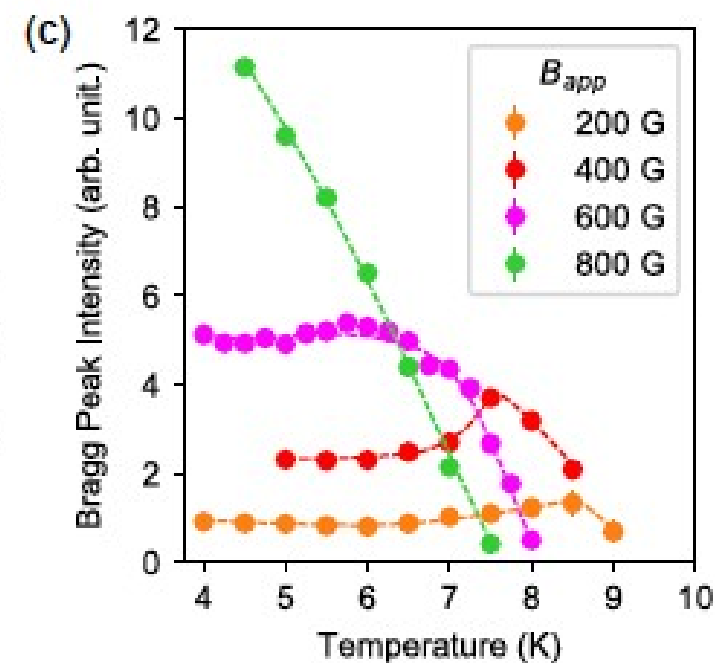
(a) Setup of a SANS experiment



(b) Manifestation of the IMS state in SANS
Dependence of the average magnetic field
as the function of temperature



(c) Manifestation of the IMS state in SANS
Bragg peak intensity as a function of temperature
for selected values of applied magnetic field



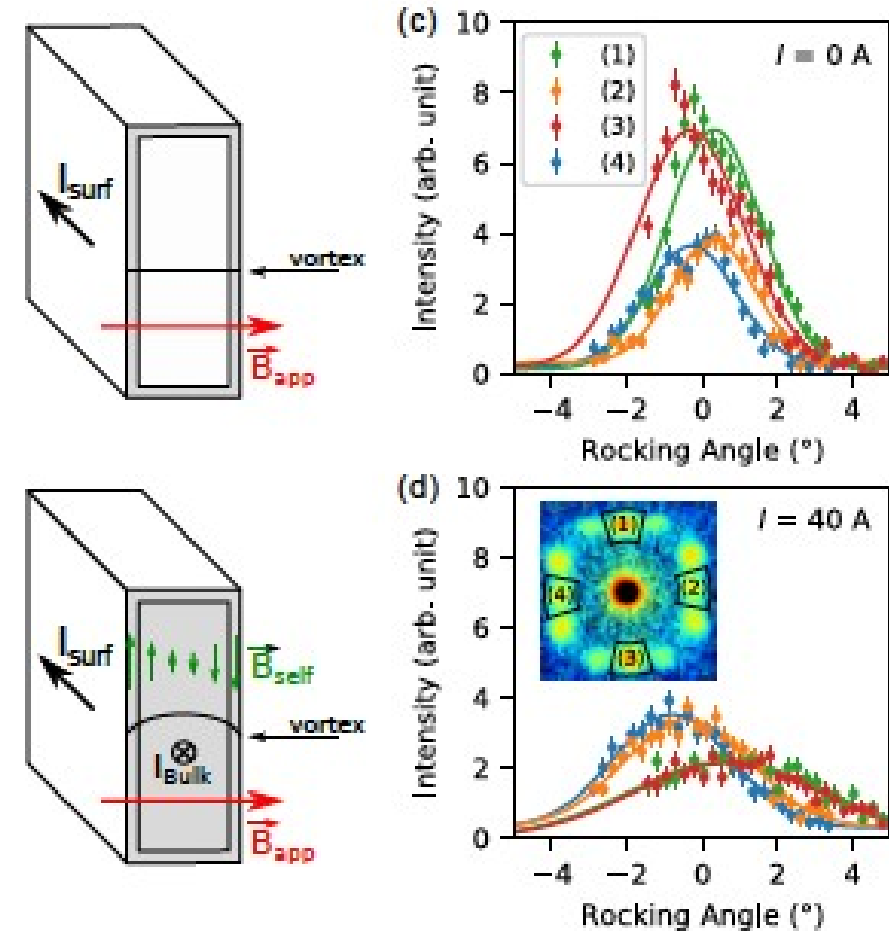
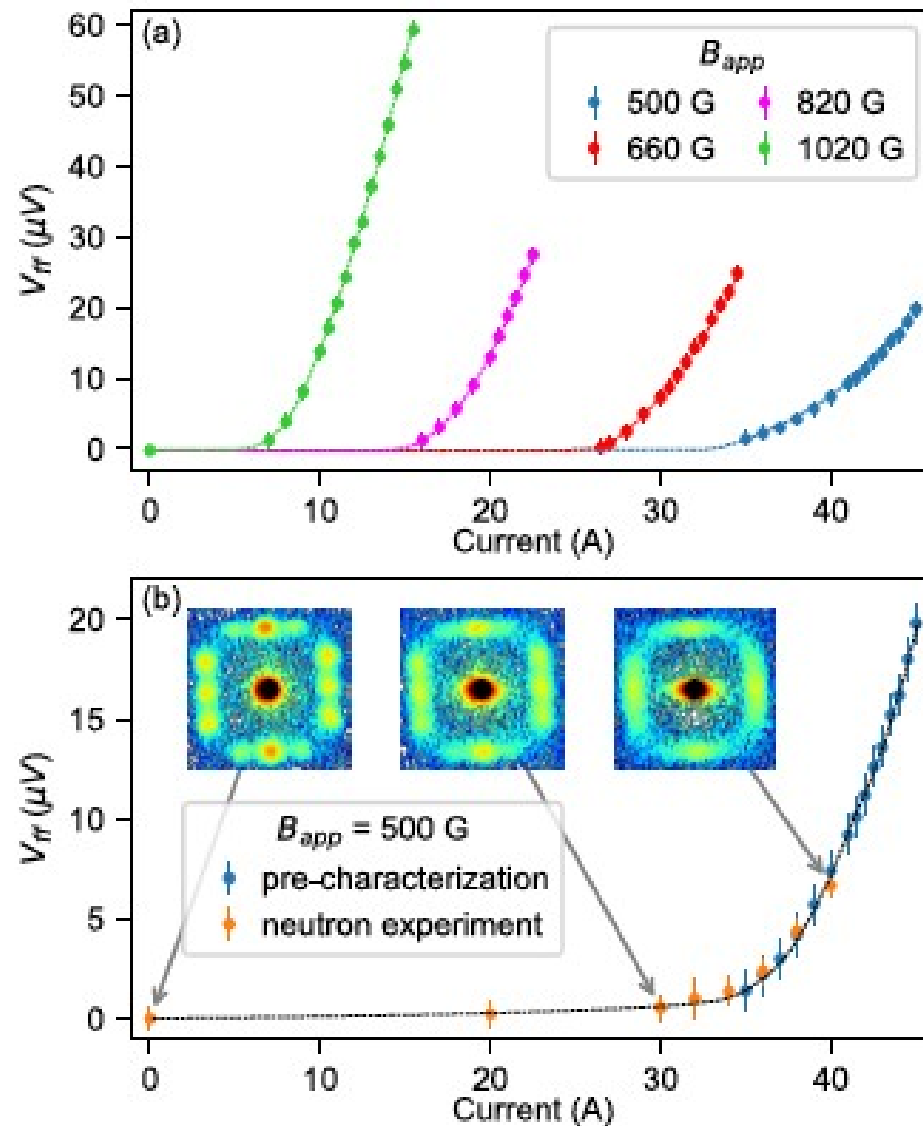
Internal field is measured by the position of Bragg peaks (for the isosceles lattice)
The presence of IMS – by the intensity of the central spot (around black circle)

T. Reimann, et al, Nat. Comm. 1, 8813 (2015)

A. Baks, et al, Phys. Rev. B 100, 064503 (2019)

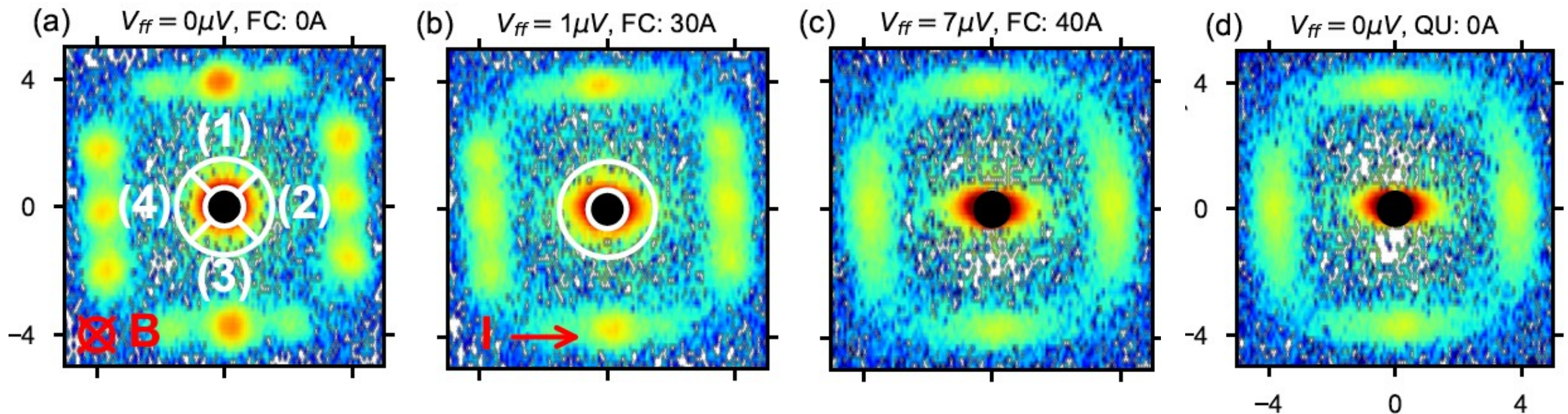
Applying external current changes SANS images

Applied current makes vortices move creating non-zero V



Applying external current changes SANS images

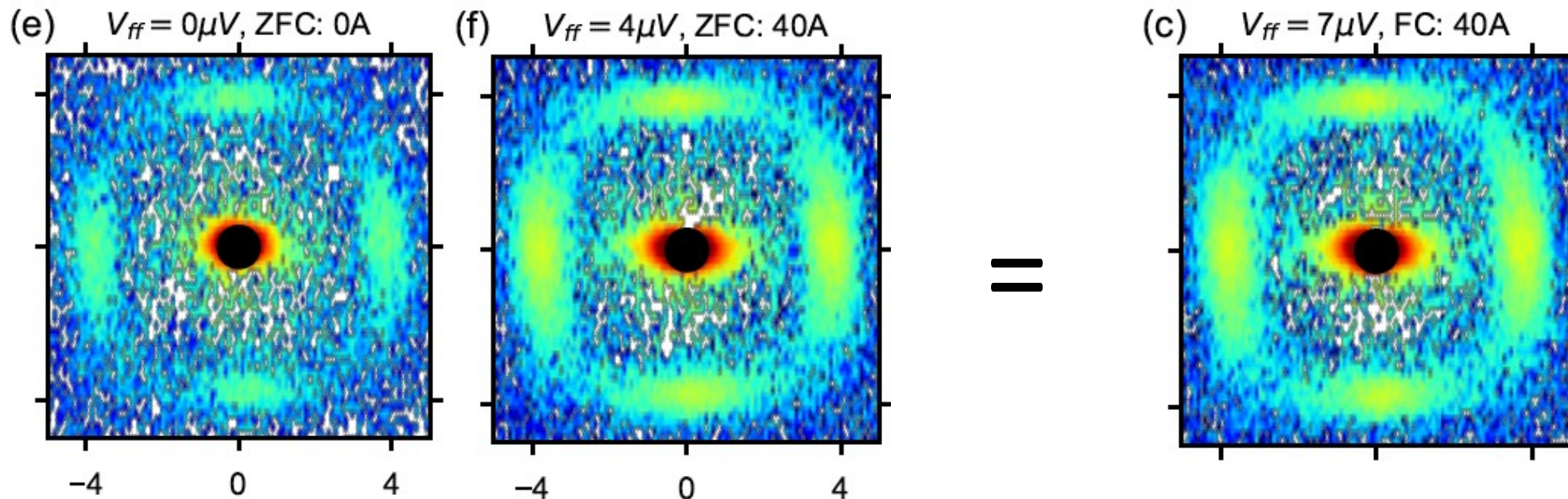
The field cooling protocol: the sample is field-cooled, then the current is applied ($J=30\text{A}, 40\text{A}$), then the current is switched off



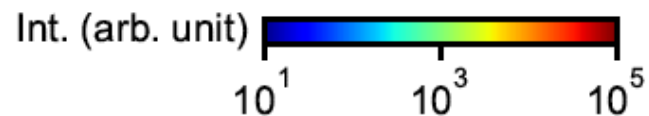
Bragg peaks are smeared, the central circle elongates in the parallel direction and remains so when the current is switched off

Applying external current changes SANS images

The field cooling protocol: the sample is zero-field-cooled, then the magnetic field is applied, then the current is applied ($J=40\text{A}$)

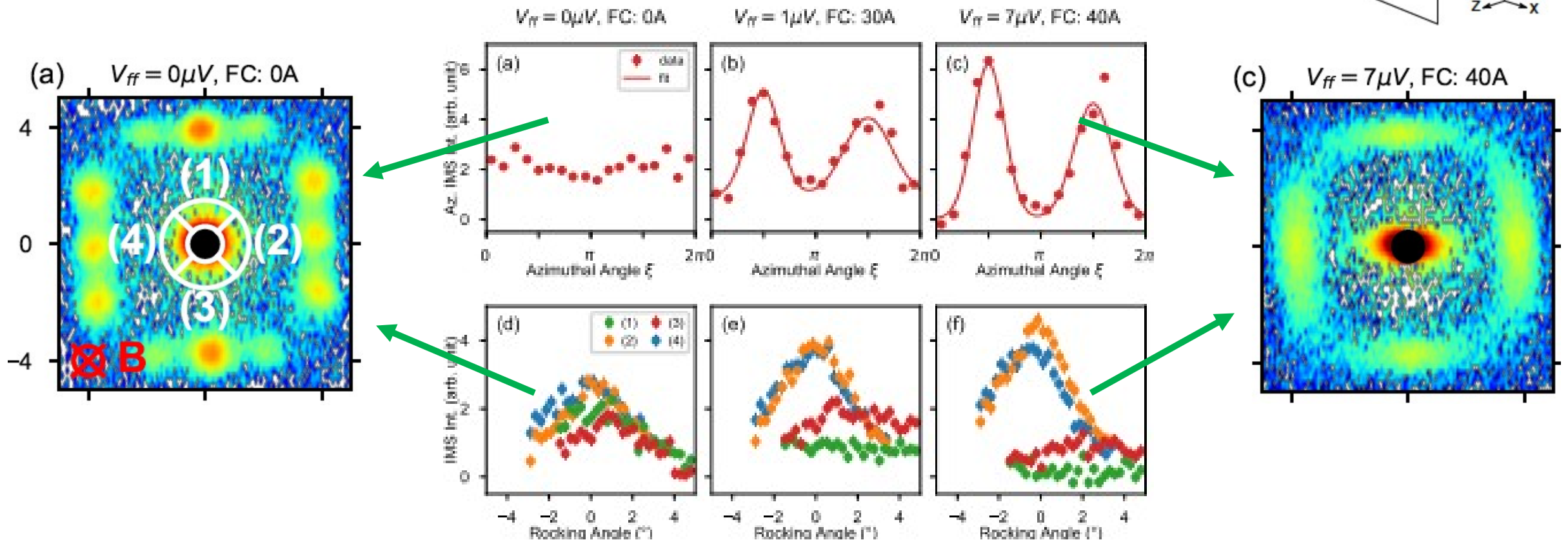
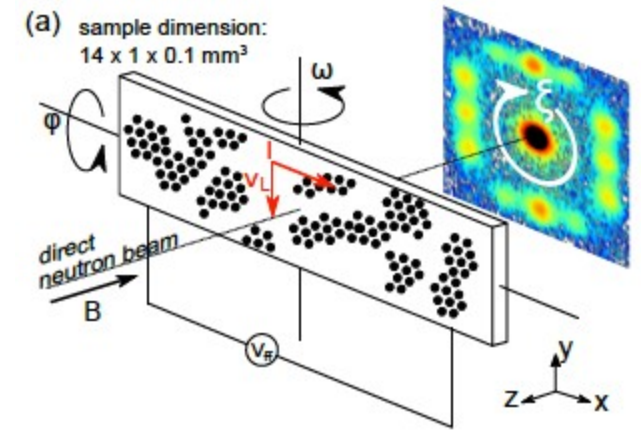


The result is the same as with the field-cooling protocol. The current induced state is independent on the preparation



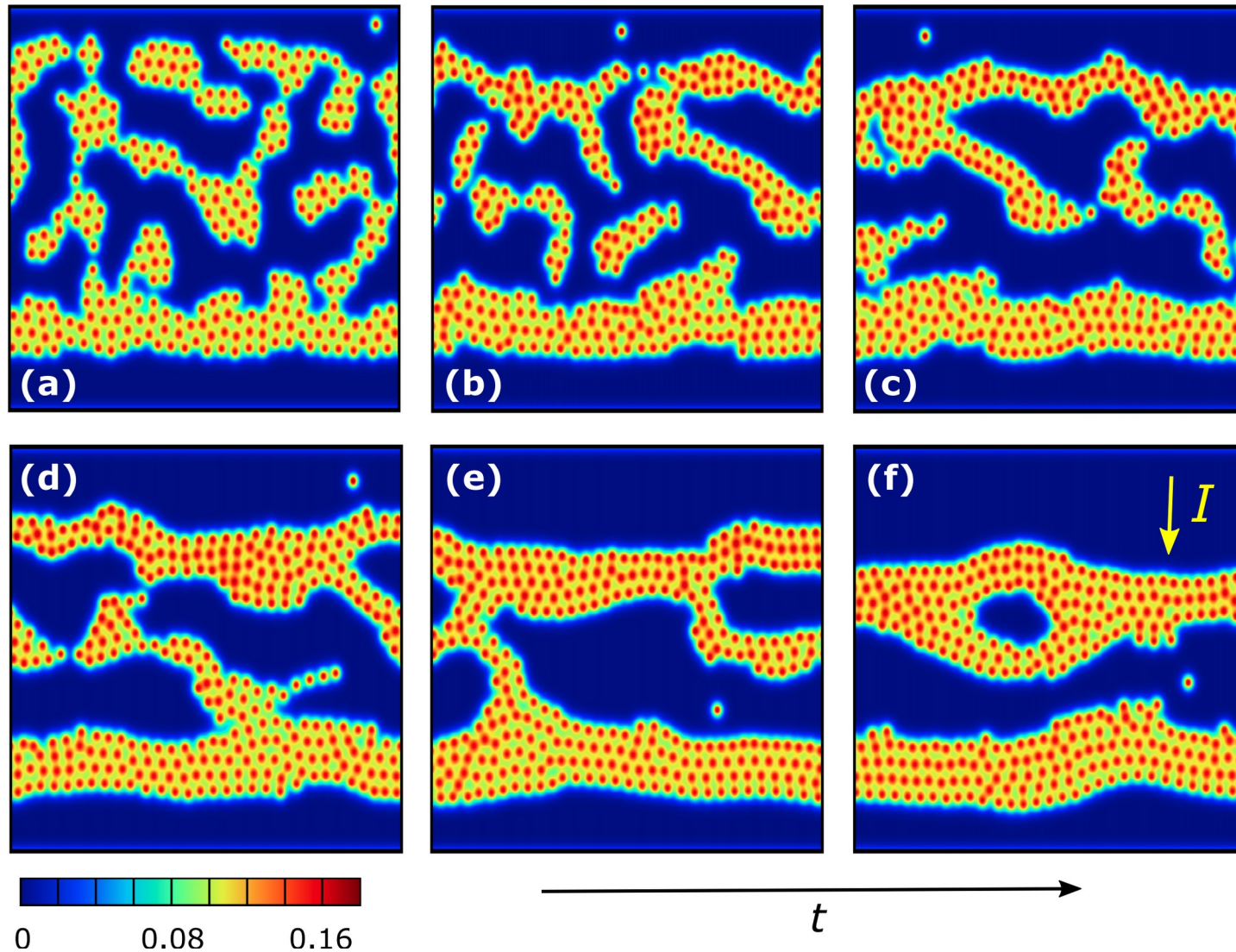
Applying external current changes SANS images

Dependence of the intensity of the IMS signal on the azimuthal angle



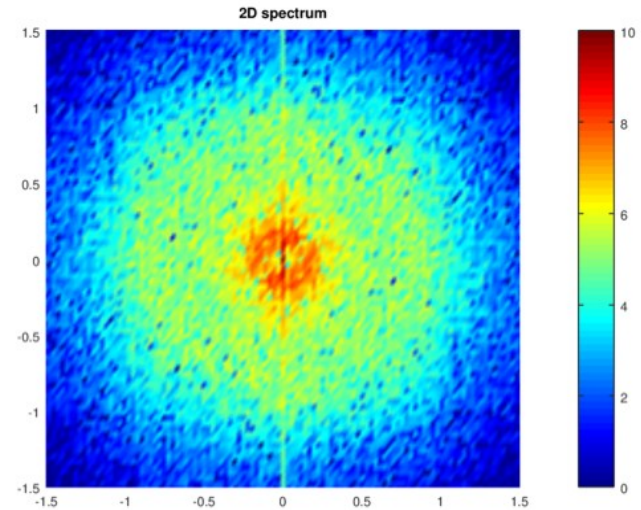
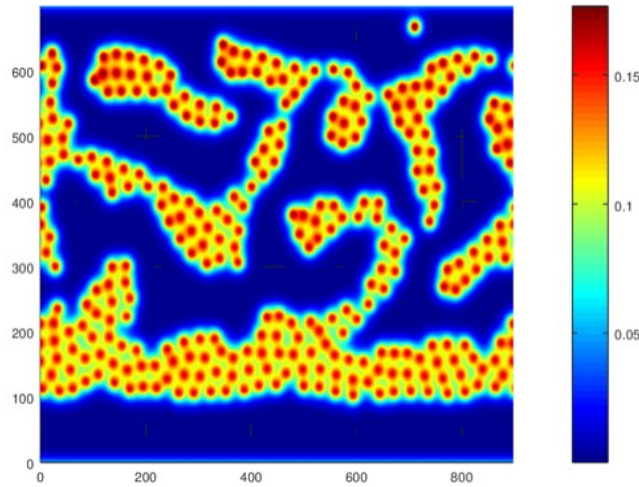
Dependence of the intensity in sectors 1-4 of the IMS signal on the rocking angles

Modelling the time evolution of the IMS

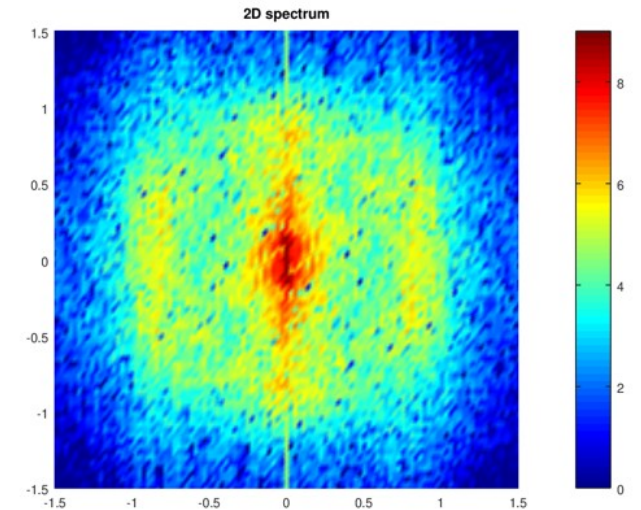
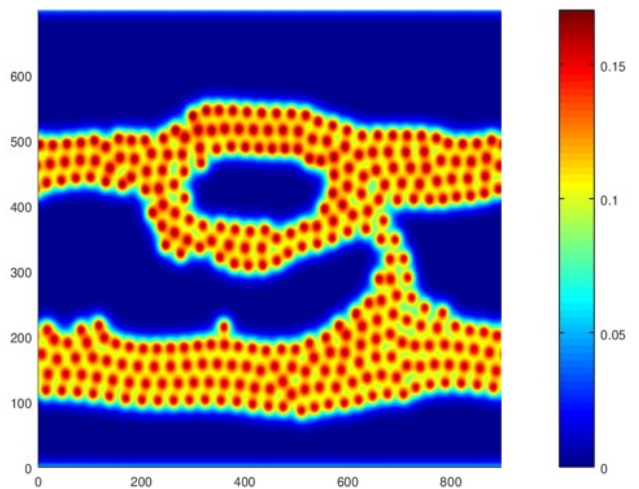


1. The evolution starts from vortex clusters (IMS)
2. The IMS structure gradually elongates in the direction perpendicular to the current
3. A superstructure of parallel vortex stripes is formed gradually

Changes in the central (IMS) peak

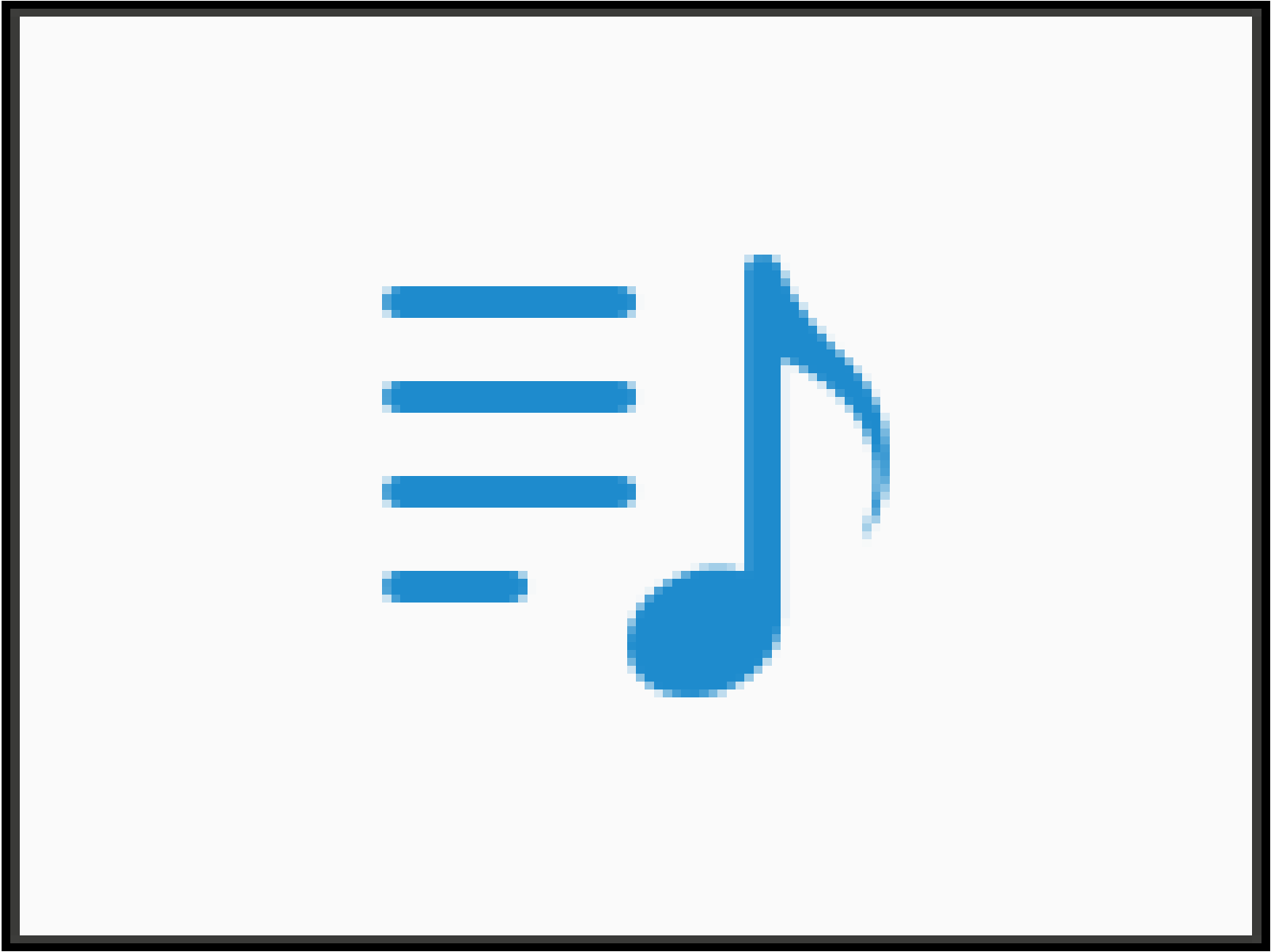


The Bragg peaks are smeared because of random orientation of vortex lattice in clusters. The similar orientation appears in the forming stripes.

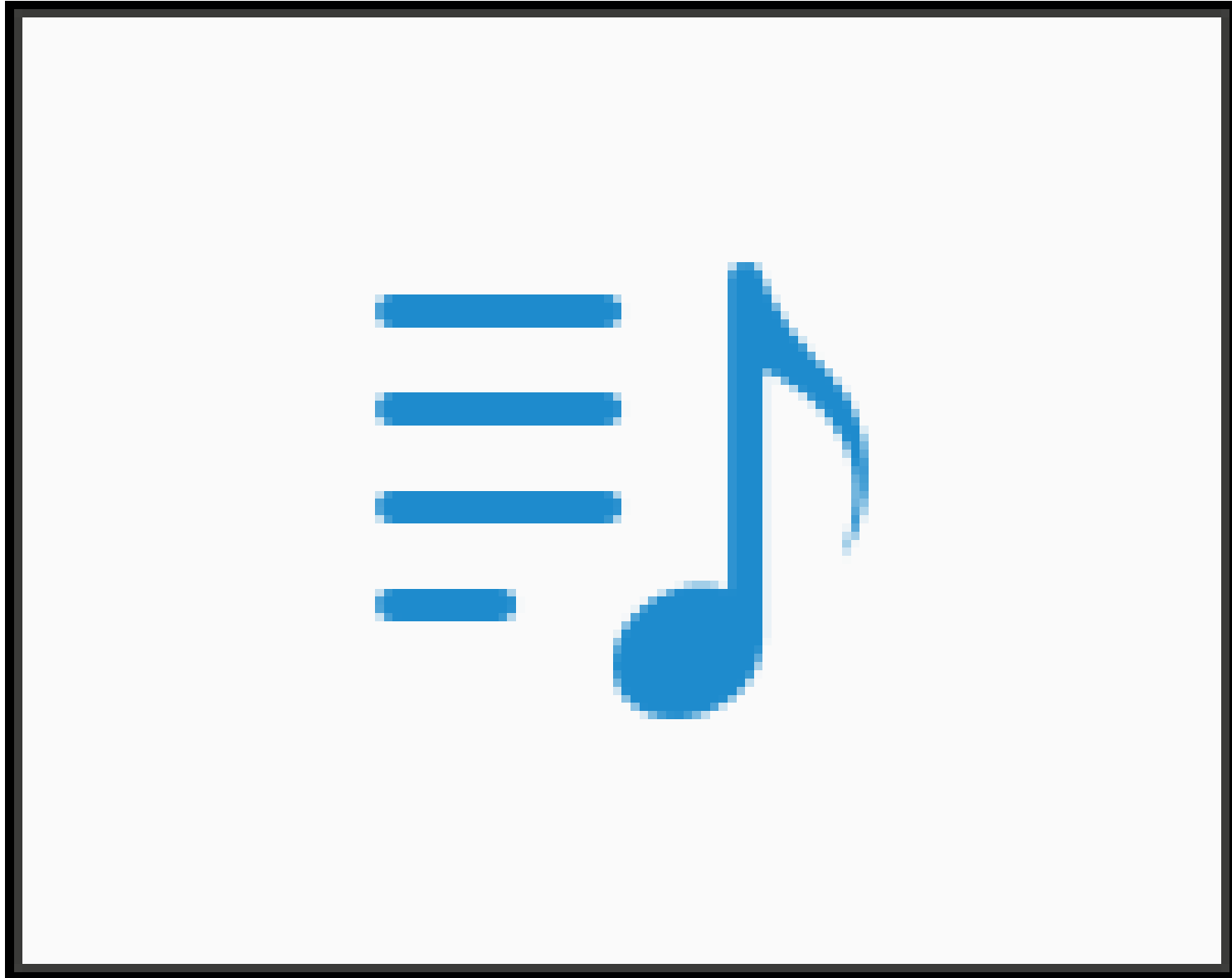


Appearance of the striped superstructure is accompanied by the elongation of the central peak in the Fourier transform

Example of the IMS time evolution

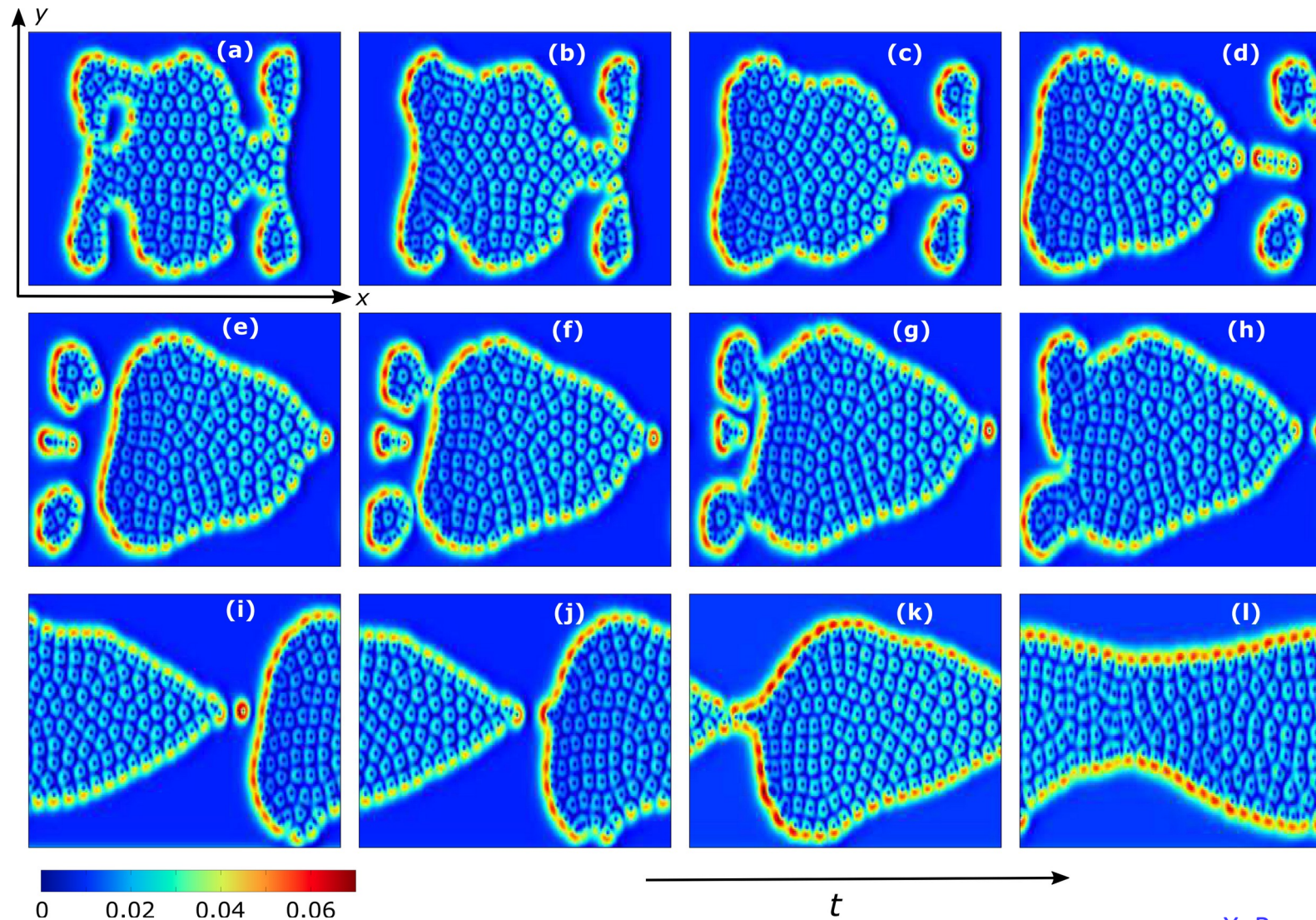


Time evolution of a cluster



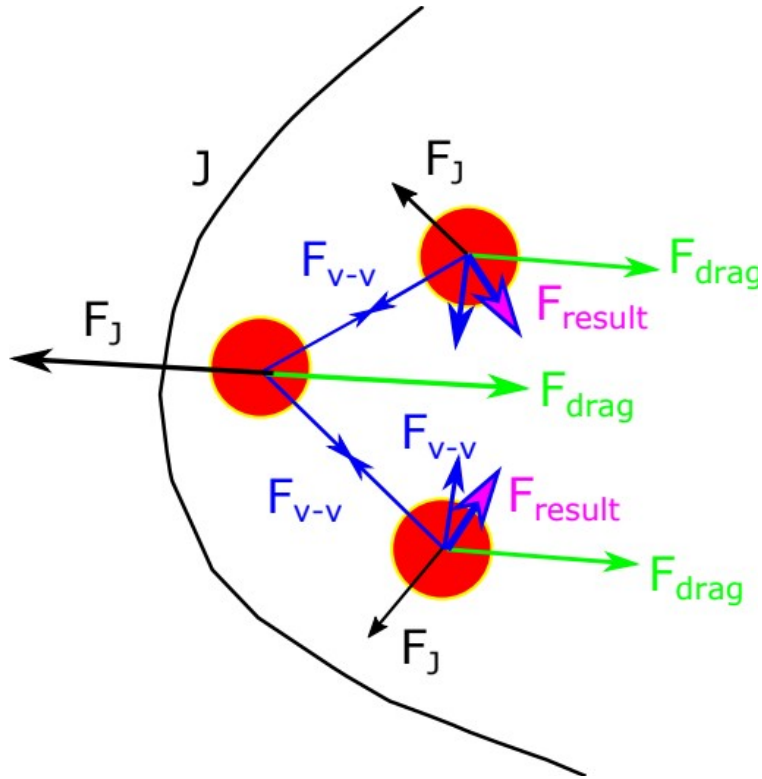
Time evolution of a cluster – snapshots.

Current density plot

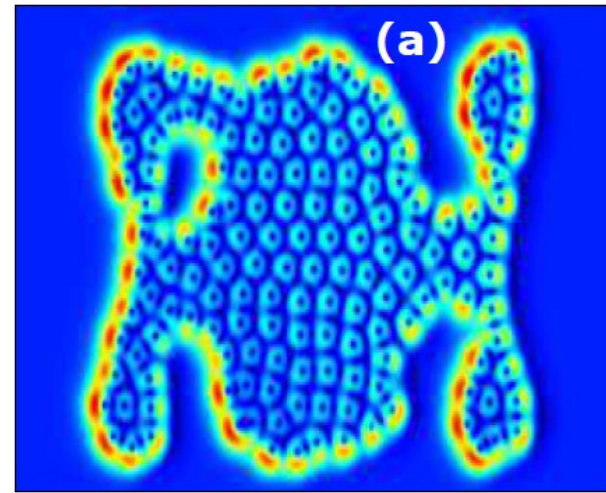


1. Voids in a cluster tend to disappear fast
2. Larger clusters move faster, they catch up smaller ones
3. Front part of a cluster tend to move faster then the rear one, the cluster elongates

Simple intuitive picture

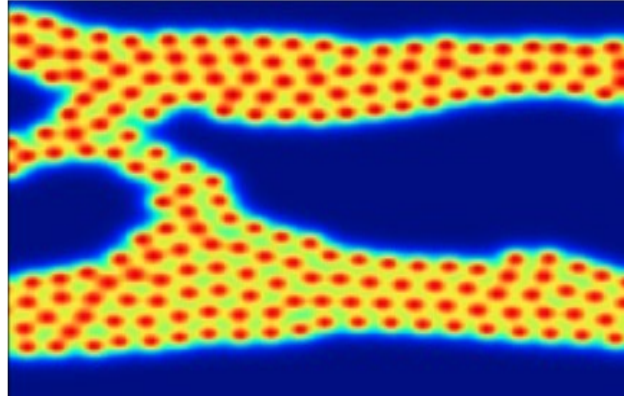


1. Current flows mostly on edges of a cluster, pulling vortices on the surface, inner vortices are least affected
2. The current density is asymmetric, flows mostly along the left edge
3. The drag force also depends on the position of the vortex
4. Single-vortex picture is inadequate, the balance of forces depends on the cluster size/shape
5. The distance between vortices holds on average, however, the structure does not – it is “soft” and changes.
6. The overall picture is like the force pulls a vortex cluster by its surface, the drag forces vortices at the rear backwards. This changes the cluster shape but not volume – it is kept because the average vortex-vortex distance is kept.



This balance of forces makes the cluster elongate in the direction of the motion. becomes eventually a “vortex river”.

Simple intuitive picture



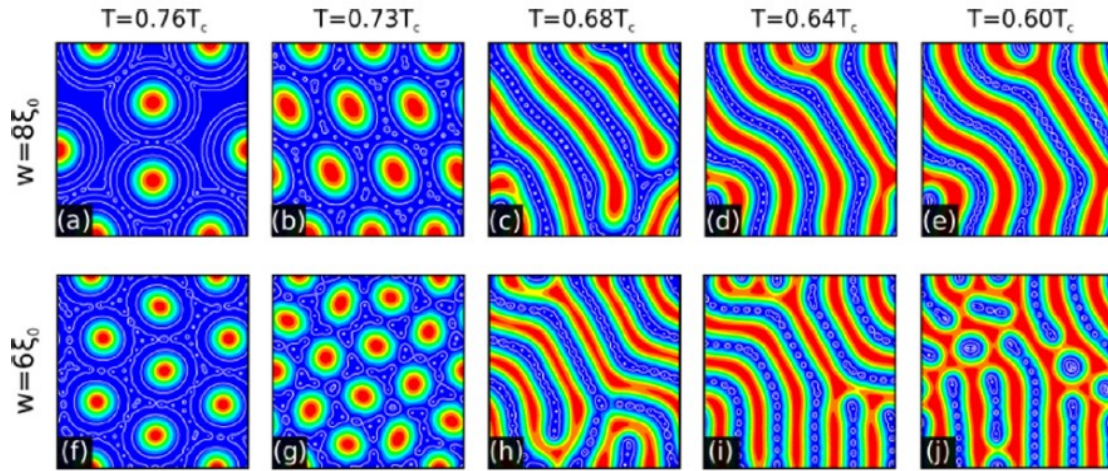
Conclusions

When the current is applied to an intertype superconductor with the intermediate mixed state, a super-structure of stripes is created perpendicular to the current flow

The elongation of IMS clusters is due to special balance of forces that depends on cluster shapes. Not a single-vortex process. Large dispersion of vortex velocities is expected.

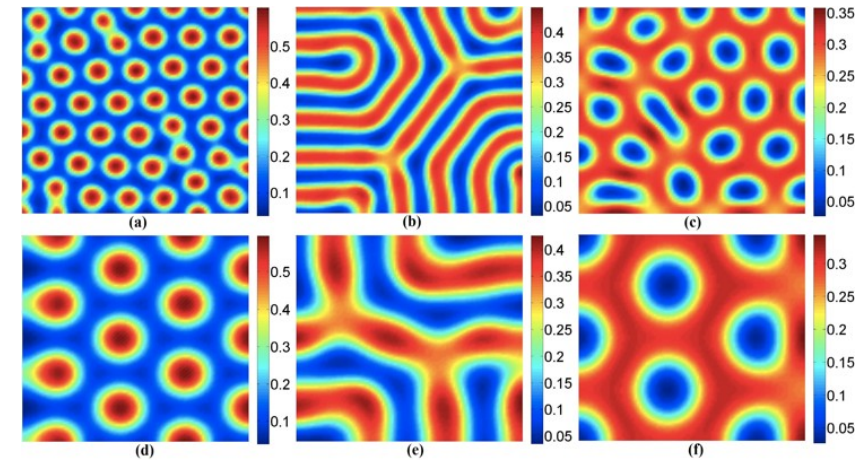
The formation of a super-structure is a robust process – not much dependent on initial conditions, protocol, magnetic field, current value, etc.

A speculative outlook – self-organized complex patterns



Spatial condensate profile in thin superconductive film

W.Y. Cordoba-Camacho, Phys. Rev. B 94, 054511 (2016)



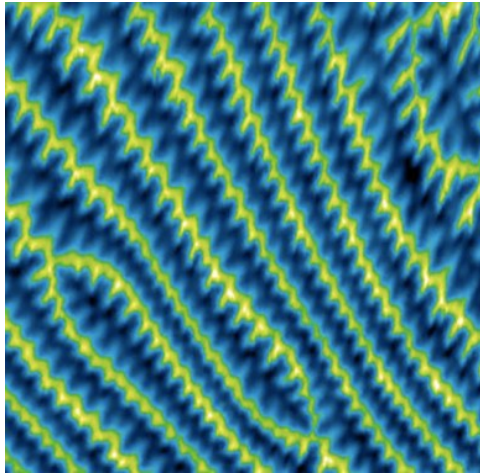
Spatial distribution of the infected in pandemic

R. M. Donlan, Emerging infect. Diseases 8, 881 (2002)

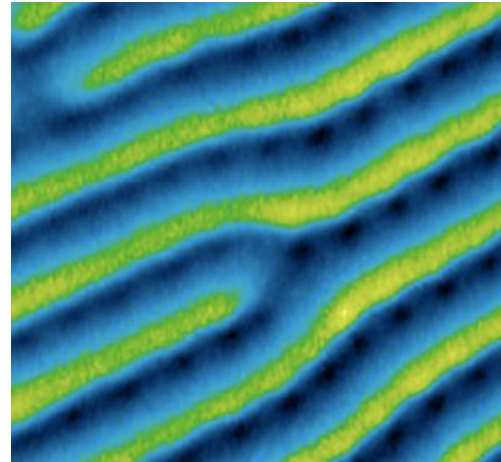
Close to the B-point of self-duality, superconductive systems reveal patterns that are remarkably similar to self-organized patterns obtained in different systems

The reason is not yet well studied, although a competition between interactions with different length scales remind the Turing mechanisms for the pattern formation. The critical B-point represents an interesting mechanism for the pattern formation, not discussed previously, based on the infinite degeneracy: all patterns exist at one point, which is unfolded into a finite interval.

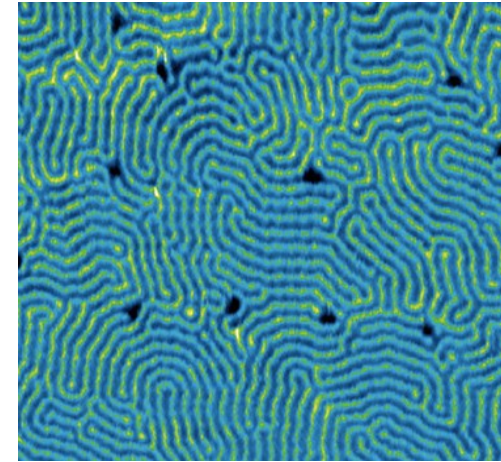
Magnetization patterns in ferromagnetic superconductors (Prof. Shanenko, tomorrow)



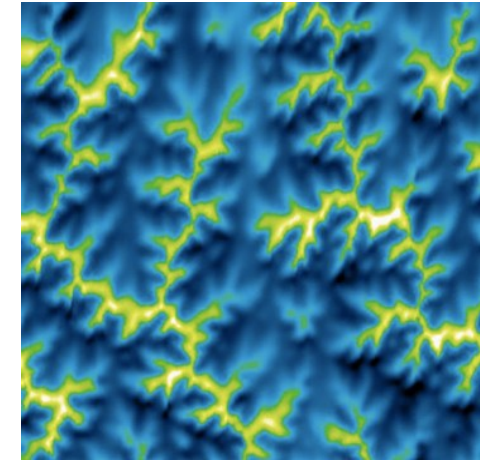
Maze patterns with the presence of Zig-Zag and Eckhaus defects.



Eckhaus instabilities of the stripe patterns



Convection-like patterns



“Ice on glass” patterns

All are spatial magnetic patterns observed in a superconducting ferromagnet $\text{EuFe}_2(\text{As}_{1-x}\text{P}_x)_2$, at temperatures $T < T_{\text{FM}} < T_c$. They appear very sensitive to the temperature and other parameters (all those patterns are obtained by varying the temperature).

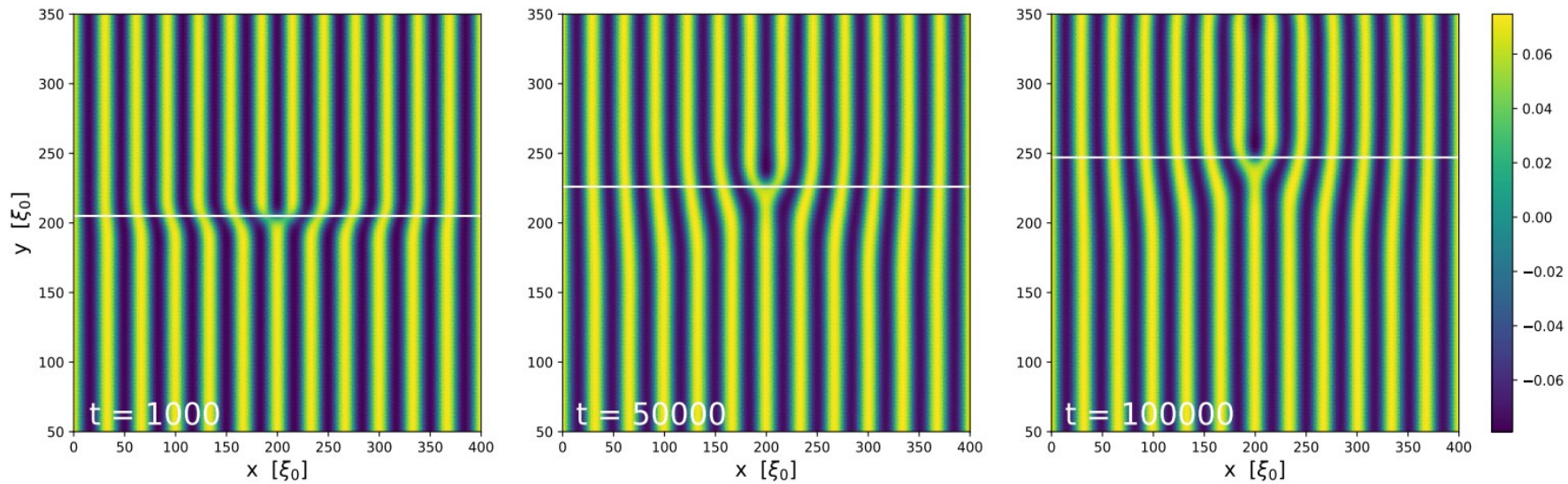
These patterns are very similar to self-organized patterns observed in chemistry, biology, etc.

Apparently, these patterns are manifestations of the interplay of two order parameters in the system.

V. Stolyarov et al., *Sci. Adv.* 2018; 4 : eaat1061

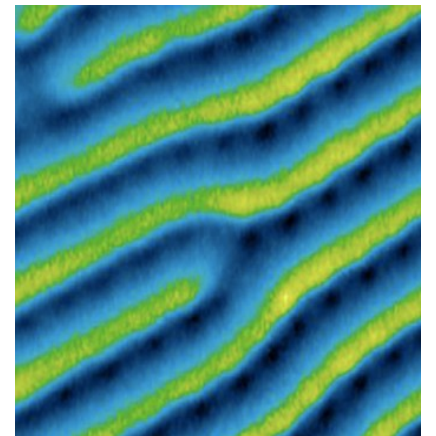
S. Grebenchuk, MS Thesis *Coexistence of superconductivity and ferromagnetism in $\text{EuFe}_2(\text{As}_{1-x}\text{P}_x)_2$ crystals*, Skolkovo Institute of Science and Technology, Moscow 2019

Patterns are dynamical !



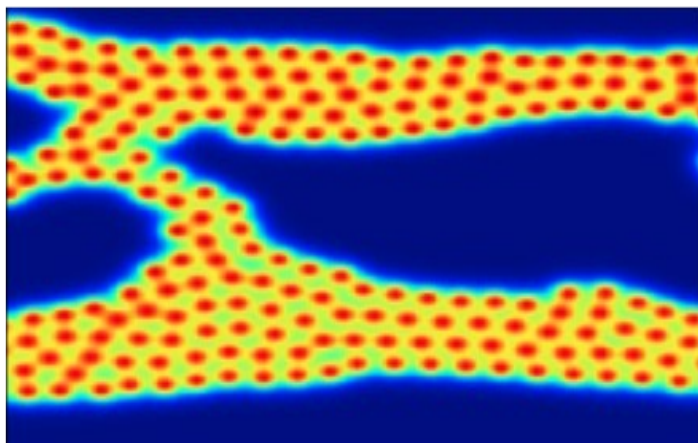
The patterns move with time - they are never completely stable

S. Kostner, A. Aladyshkin, V. Stolyarov, work in progress



Eckhaus instabilities

Thank you for your attention



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N.V. Orlova, A.A. Shanenko, M.V. Milosevic, F.M. Peeters, A. Vagov and V. M. Axt, *Ginzburg- Landau theory for multiband superconductors: Microscopic derivation*, Phys. Rev. B 87, 134510 (2013)

A. Vagov, A.A. Shanenko, M.V. Milosevic, V.M. Axt, and F.M. Peeters, *Two-band superconductors: Extended Ginzburg-Landau formalism by a systematic expansion in small deviation from the critical temperature*, Phys. Rev. B 86, 144514 (2012)

A.V. Vagov, A.A. Shanenko, M.V. Milosevic, V.M. Axt, and F.M. Peeters, *Extended Ginzburg- Landau formalism: systematic expansion in small deviation from the critical temperature*, Phys. Rev. B 85, 014502 (2012)

A.A. Shanenko, M.V. Milosevic, F.M. Peeters and A.V. Vagov, *Extended Ginzburg-Landau Formalism for Two-Band Superconductors*, Phys. Rev. Lett. 106, 047005 (2011)

Theoretical simulations. The model

Two-component GL model

$$f = \sum_{\nu=1,2} \left(\frac{1}{2m_{\nu}} |\mathbf{D}\Psi_{\nu}|^2 + \alpha_{\nu} |\Psi_{\nu}|^2 + \frac{\beta_{\nu}}{2} |\Psi_{\nu}|^4 \right) - \Gamma \{ \Psi_1^* \Psi_2 + \Psi_1 \Psi_2^* \} + \frac{\mathbf{B}^2}{8\pi}$$

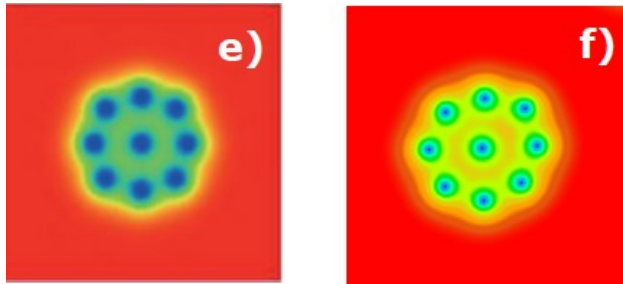
$$\eta D_t \psi_1 = \mathbf{D}^2 \psi_1 - (\chi_1 - |\psi_1|^2) \psi_1 - \gamma \psi_2,$$

$$\eta D_t \psi_2 = \frac{1}{\alpha} \mathbf{D}^2 \psi_2 - (\chi_2 - |\psi_2|^2) \psi_2 - \frac{\beta_2}{\beta_1} \gamma \psi_1,$$

$$\kappa_1^2 \nabla \times \nabla \times \mathbf{A} = \frac{1}{\kappa_1^2} \Re[\psi_1 \mathbf{D}\psi_1^*] + \frac{\alpha}{\kappa_2^2} \Re[\psi_2 \mathbf{D}\psi_2^*],$$

E. Babaev et al., Phys. Rev. Lett. 105, 067003 (2010)

R.M. da Silva et al., Sci. Rep. 5, 12695 (2015)



2-band: E. Babaev et al., Type-1.5 Superconductivity.
In Superconductors at the Nanoscale,
De Gruyter, 2017; p 133

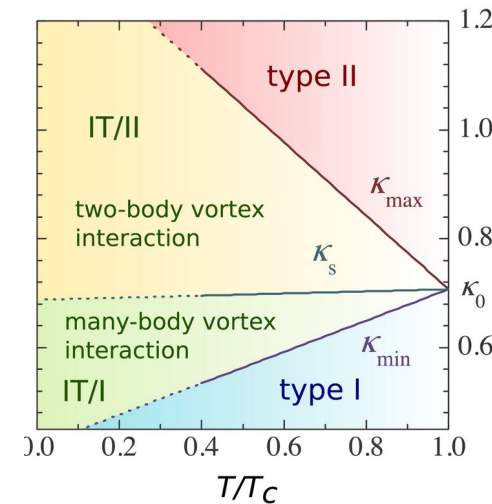
1-band: A. Vagov et al., Commun. Phys. 3, 58 (2020)

The reason: 1-band beyond-GL calculations and 1-band GL-like models give qualitatively similar IMS configurations

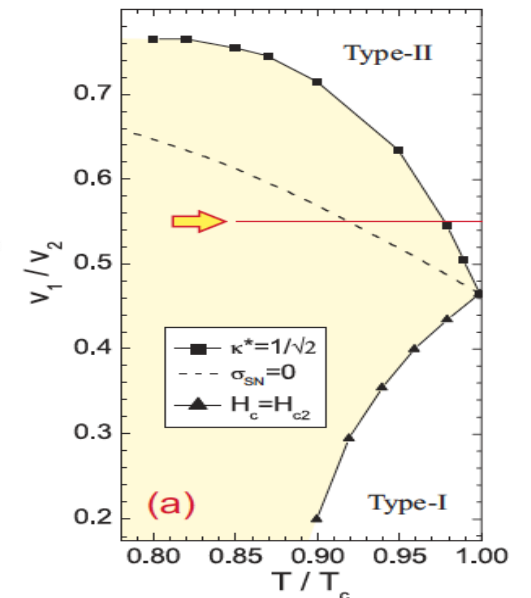
1. non-monotonic vortex interactions
2. increased role of multi-vortex interactions
3. same clustering and phase diagram

GL-like models easy to calculate

1-band



2-band



2-band: R.M. da Silva et al., Sci. Rep. 5, 12695 (2015)

1-band: S. Wolf et al, Phys. Rev. B 96, 144515 (2017)